

$$f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0$$

$$\rightarrow \frac{-2x + 1}{x(x-1)} \geq 0$$

تعیین علامت

x	0	$\frac{1}{2}$	1
$+$	$-$	$+$	$-$

$$\rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$$

$$f(x) = \frac{1}{x+1} - \frac{x}{x-1} \rightarrow \frac{1}{x+1} + \frac{1}{x-1} \rightarrow \frac{x-1}{x(x-1)} + \frac{x+1}{x(x-1)} \rightarrow \frac{x-1+x+1}{x(x-1)} \rightarrow \frac{2x}{x(x-1)} \rightarrow \frac{2}{x-1} \neq 0 \rightarrow x \neq 1$$

$$\rightarrow D_f = \mathbb{R} - \{1\} = (-\infty, 1) \cup (1, +\infty)$$

$$f(x) = \sqrt{\left(\frac{1}{x}\right)^2 - 9(x^2 - 4^x)} \rightarrow \left(\frac{1}{x}\right)^2 - \left(\frac{1}{x}\right)^2 (x^2 - 4^x) \geq 0 \rightarrow \frac{1}{x^2} (1 - (x^2 - 4^x)) \geq 0 \rightarrow 1 - x^2 + 4^x \geq 0$$

تعیین علامت

x	0	1	2
$+$	$-$	$+$	$+$

$$\rightarrow D_f = (-\infty, 0) \cup [1, +\infty) \rightarrow D_f = (-\infty, 0) \cup [1, +\infty)$$

$$\sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow \begin{cases} ① x-1 \geq 0 \rightarrow x \geq 1 \\ ② \sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow 3 - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10 \end{cases}$$

$$\rightarrow D_f = ① \cap ② = [1, 10]$$

$$f(x) = \frac{\log_2(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow \begin{cases} ① x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x < -1 \text{ یا } x > 2 \\ ② x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1 \text{ یا } x \leq -1 \\ ③ \sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1 \rightarrow \text{مخرج نباید صفر شود} \end{cases}$$

زیر رادیکال همواره مثبت است پس مخرج هیچ وقت صفر نمی شود پس این بشرط همواره برقرار است.

$$\rightarrow D_f = ① \cap ② = (-\infty, -1) \cup [2, +\infty)$$

$$f(x) = \sqrt{3 + a^2 x - x^2} \rightarrow 3 + a^2 x - x^2 \geq 0 \rightarrow x^2 - a^2 x - 3 = 0$$

نقاط انتهایی دامنه یعنی x_1 و x_2 رابطه های عبارت زیر رادیکال هستند.

$$\rightarrow x^2 - a^2 x - 3 = 0 \rightarrow \begin{cases} p = -3 = (-a^2) \times b \rightarrow b = \frac{3}{a^2} \\ S = -ca = b - 3 \rightarrow a = \frac{1}{p} \end{cases}$$

$$\rightarrow a + b = \frac{1}{p} + \frac{3}{p} = \frac{4}{p} = 1$$

$$\rightarrow a + b = 1$$

$$g(x) = \sqrt{f(x) - x} \rightarrow f(x) - x \geq 0 \rightarrow \begin{cases} ① x \geq 1 \rightarrow 3x - 2 - x \geq 0 \rightarrow 2x - 2 \geq 0 \rightarrow x \geq 1 \\ ② x < 1 \rightarrow 2x + 3 - x \geq 0 \rightarrow x + 3 \geq 0 \rightarrow x \geq -3 \end{cases}$$

درجه ۱ است

$$\rightarrow D_f = ① \cup ② \rightarrow D_f = [-3, +\infty)$$

$$f(x) = x^p - 1 \rightarrow f(x) = x^p - 1 = (x^p - 1) + 1 - 1 \rightarrow f(x) = (x^p - 1) + 1 - 1 \rightarrow f(x) = (x^p - 1) + 1 - 1$$

$$\rightarrow f(\sin^p x + \cos^p x) \rightarrow x = \sin^p x + \cos^p x = \sqrt[p]{p} + 1$$

$$f(\sqrt[p]{p} + 1) = (\sqrt[p]{p} + 1)^p - 1 = (\frac{p}{p} + 1)^p - 1 = \frac{p}{p} - 1 = \frac{1}{p}$$

$$\rightarrow f(\sin^p x + \cos^p x) = \frac{1}{p}$$

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$$f(x) = \frac{x+1}{x-1} \rightarrow f(a) = \frac{a+1}{a-1} \rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a+1} = \frac{2a}{2} = a$$

$$\rightarrow f(f(a)) = \frac{a}{1} \rightarrow a-1 \neq 0 \rightarrow a \neq 1$$

$$\rightarrow a \in \mathbb{R} - \{1\} \subseteq a \neq 1 \subseteq (-\infty, 1) \cup (1, +\infty)$$

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$$f(g(a)) = -f \rightarrow f(\sqrt{a^2-1}) = -f \rightarrow \frac{\sqrt{a^2-1}}{t-\sqrt{a^2-1}} = -f \rightarrow t - \sqrt{a^2-1} = -f \rightarrow t - \sqrt{a^2-1} + f = 0$$

$$\rightarrow t = 1 \rightarrow \sqrt{a^2-1} = 1 \rightarrow a^2-1 = 1 \rightarrow a^2 = 2 \rightarrow a = \pm\sqrt{2}$$

$$\rightarrow a = \pm\sqrt{2}$$

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$$f(x+y) = x^2 - \omega x + y \rightarrow \begin{cases} f(x) \rightarrow x^2 - \omega x = f \rightarrow x = 1 \rightarrow f(x) = 1 - \omega + y = 1 \\ f(y) \rightarrow y^2 - \omega y = -f \rightarrow y = -y \rightarrow f(y) = y^2 - \omega y + y = y^2 \end{cases}$$

$$\rightarrow f(x) \times f(y) = 1 \times 1 = 1$$

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$$f(\cos^2 x + \omega \cos x) = \tan^2 x \rightarrow f \cos^2 x + \omega \cos x = \tan^2 x = \frac{1}{\cos^2 x}$$

$$t = \cos x \rightarrow f t^2 + \omega t = \frac{1}{t^2} \rightarrow f t^2 + \omega t - \frac{1}{t^2} = 0 \rightarrow \lambda t^2 + 10 t - 1 = 0$$

$$\rightarrow t = \frac{1}{p} \rightarrow f(\frac{1}{p}) = \tan^2 x \rightarrow \tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x}$$

$$\rightarrow \tan^2 x = \frac{1 - \frac{1}{p^2}}{\frac{1}{p^2}} = \frac{p^2 - 1}{p^2} \times p^2 = p^2 - 1 \rightarrow f(p, \omega) = p^2 - 1$$

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