

(الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{x^2-2x+1-x^2}{x^2-x} > 0 \Rightarrow \frac{1-2x}{x(x-1)} > 0$ (۱)

(ب) $f(x) = \frac{1}{x+1} - \frac{2}{x} \Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

$\frac{1}{x+1} - \frac{2}{x} = \frac{x - 2x - 2}{x(x+1)} = \frac{-x-2}{x(x+1)}$

$D_f = \mathbb{R} - \{-2, -\frac{5}{4}, -1, 0, 1\}$

$\begin{cases} x(x+1) \neq 0 \Rightarrow x \neq \{0, -1\} \\ (x-1)(x+2) \neq 0 \Rightarrow x \neq \{-2, 1\} \\ 4x+5 \neq 0 \Rightarrow x \neq \{-\frac{5}{4}\} \end{cases}$

(الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right)(3^x - 4^x)}$ (۲)

(ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt[4]{x-1} \rightarrow x \geq 1$

$3 - \sqrt[4]{x-1} \geq 0 \Rightarrow \sqrt[4]{x-1} \leq 3 \Rightarrow x \leq 12$

$\Rightarrow D_f = [1, 12]$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow \frac{-1}{2} < x < 2$

$\sqrt{x^2-1} + 1 \rightarrow \sqrt{x^2-1} + 1 \neq 0 \checkmark \rightarrow$ همیشه برقرار

$\sqrt{x^2-1} \geq 0 \Rightarrow x \neq (-1, 1)$

$D_f = (-\infty, -1) \cup (2, +\infty)$ (۳)

(۴) یعنی -2 ریشه های $(-x^2 + ax + 2)$ هستی

$-2 - 2a + 2 = 0 \Rightarrow 2a = -1 \rightarrow a = -\frac{1}{2}$

$(-2)(b) = -2 \rightarrow b = \frac{2}{2} = 1$

$a+b = \frac{2}{2} - \frac{1}{2} = 1$

if $x \geq 1 \rightarrow \sqrt{\frac{2x-2-x}{2x-2}} \rightarrow \frac{2(x-1)}{2x-2} \geq 0 \rightarrow x \geq 1$

if $x < 1 \rightarrow \sqrt{\frac{2x+3-x}{x+3}} \rightarrow \frac{x+3}{x+3} \geq 0 \rightarrow x \geq -3$

$D_f = [-3, +\infty)$ (۵)

$\sin \frac{\pi}{\sqrt{2}} + \cos \sqrt{2}\pi = \frac{\sqrt{2}}{\sqrt{2}} + 1 = 2$ (۶)

$f(x) = 2x^2 - 4x = 2(x-1)^2 - 2$

$\Rightarrow f\left(\frac{\sqrt{2}}{\sqrt{2}} + 1\right) = 2\left(\frac{\sqrt{2}}{\sqrt{2}} + 1 - 1\right)^2 - 2 = 2\left(\frac{\sqrt{2}}{\sqrt{2}}\right)^2 - 2 = \frac{2}{2} - 2 = -1$

$$f(a) = \frac{a+1}{a-1} \rightarrow a \neq 1 \rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a \quad (V)$$

برای همه مقادیر $a \neq 1$ برقرار است

$a=1$ برقرار است $\leftarrow \mathbb{R} - \{1\}$

$$f(g(a)) = -f \xrightarrow{g(a)=x} x^2 - f x = -f \Rightarrow x^2 - f x + f = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1 \quad (A)$$

$$g(a) = 1 \rightarrow \sqrt{a^2 - 1} = 1 \Rightarrow a = \sqrt{2}$$

$$f = x + y \Rightarrow x = 1 \rightarrow f(f) = f(1+1) = 1 - 2 + 4 = 3 \quad (9)$$

$$-y = x + y \rightarrow x = -y \Rightarrow f(-y) = f(-y+y) = y^2 - 2(-y) + y = 3y$$

$$f(f) \times f(-y) = 3 \times 3y = 9y$$

$$f \cos x + 2 \cos x = 1 \Rightarrow f \cos x + 2 \cos x - 1 = 0 \quad (10)$$

$$\Rightarrow \cos x + 2 \cos x - 1 = 0 \rightarrow (\cos x + 1)(\cos x - 1) = 0$$

$$\Rightarrow \cos x = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} \checkmark$$

$$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}$$

$$\Rightarrow \tan\left(\frac{\pi}{3}\right) = (\sqrt{3})^1 = \sqrt{3}$$