

نام و نام خانوادگی: امیرحسین محمدی پاسخنامه تشریحی تکلیف شماره ۵۰ کلاس: یازدهم ۲۰ افزاینده‌ی مایل نیستی

الف) $f(x) = \sqrt{\frac{x-1}{2x} - \frac{x}{x-1}} \rightarrow \sqrt{\frac{x^2-2x+1-x^2}{2x(x-1)}} \rightarrow \sqrt{\frac{1-2x}{x(x-1)}}$ ریشه ها: ۱ و ۰ $\frac{1}{x} + \frac{1}{x-1} = 0 \rightarrow \frac{1}{x} = -\frac{1}{x-1} \rightarrow x = -x+1 \rightarrow 2x=1 \rightarrow x=\frac{1}{2}$

ب) $\frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}} = 0 \rightarrow \frac{3}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow 3x+2x+2x-1 \rightarrow 8x+2 \neq 0 \rightarrow x \neq -\frac{1}{4}$

جواب: $\mathcal{D} = (-\infty, 0) \cup (\frac{1}{2}, 1) \rightarrow$ ۴

الف) $f(x) = \sqrt{((\frac{1}{x})^x - 9)(32 - 4x^2)} \rightarrow ((\frac{1}{x})^x - 9)(32 - 4x^2) \geq 0 \rightarrow (\frac{1}{x})^x - 9 \geq 0 \rightarrow 3^2 \geq 3^x \rightarrow -x \geq 2 \rightarrow x \leq -2$

جواب: $\mathcal{D} = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow x-1 \geq 0 \rightarrow x \geq 1$ و $\sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10$

جواب: $\mathcal{D} = [1, 10]$

$f(x) = \frac{\sqrt{x^2-x-2}}{\sqrt{x^2-1}+1} \rightarrow x^2-x-2 \geq 0 \rightarrow (x-2)(x+1) \geq 0$ -۱ ۲ $\frac{-1}{2} \pm \frac{\sqrt{1+4}}{2} = \frac{-1 \pm 2}{2} = \frac{1}{2}, -1$

$\rightarrow \mathcal{D} = (-\infty, -1) \cup (2, +\infty)$ I و $\sqrt{(x-1)(x+1)} + 1 \geq 0 \rightarrow (x-1)(x+1) \geq -2 \rightarrow x \in \mathbb{R}$ II

$\rightarrow \mathcal{D} = (-\infty, -1) \cup (2, +\infty)$ ۳

$f(x) = \sqrt{3+ax-x^2}$ و $\mathcal{D} = [-2, b]$ ریشه ها $\rightarrow 3+ax-x^2 \geq 0 \rightarrow x = -2$ و $x = b$

$3+ax-x^2 = -(x+2)(x-b) \rightarrow a = b-2$ و $2b = 3 \rightarrow b = \frac{3}{2}$

$a = \frac{3}{2} - 2 = -\frac{1}{2} \rightarrow a+b = 1$ ۴

$f(x) = \begin{cases} 3x-2 \geq 1 \\ 3x+2 < 1 \end{cases}$ و $g(x) = \sqrt{f(x)-x} \rightarrow f(x)-x \geq 0$

$\rightarrow x \geq 1 \rightarrow 3x-2-x \geq 0 \rightarrow 2x-2 \geq 0 \rightarrow x \geq 1$

$\rightarrow x < 1 \rightarrow 3x+2-x \geq 0 \rightarrow 2x+2 \geq 0 \rightarrow x \geq -1$

جواب: $\mathcal{D} = [-1, 1]$ ۵

$$f(x) = x^2 - 1 \quad \sin \frac{x}{\sqrt{2}} = \frac{\sqrt{x}}{\sqrt{2}} \quad \cos 2x = 1$$

$$f(\sin \frac{x}{\sqrt{2}} + \cos 2x) = 1 \left(\frac{\sqrt{x}}{\sqrt{2}} + 1 \right)^2 + 1 \left(\frac{\sqrt{x}}{\sqrt{2}} + 1 \right) = 1 \left(\frac{x}{2} + 1 + \sqrt{x} \right) + \left(\sqrt{x} + 1 \right)$$

$$\frac{x}{2} + 1 - \varepsilon = \frac{x}{2} - 1 = \frac{-1}{2} \rightarrow -\frac{1}{2}$$

$$f(x) = \frac{x+1}{x-1} \quad , \quad f(f(a)) = a$$

$$f(f(a)) = \frac{f(a)+1}{f(a)-1} \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a \rightarrow f(f(a)) = \frac{2a}{2} = a$$

$$\rightarrow a \in \mathbb{R} - \{1\} \rightarrow \text{جواب: } \text{كل } a \text{ حقيقي غير يساوي 1}$$

$$f(x) = x^2 - 1 \quad g(x) = \sqrt{x^2 - 1} \quad f(g(a)) = -\varepsilon \quad a = ?$$

$$g(a) = t \rightarrow t^2 - 1t = -\varepsilon \rightarrow t^2 - \varepsilon t + \varepsilon = 0 \rightarrow (t-1)^2 = 0$$

$$\rightarrow t = 1 \rightarrow \sqrt{a^2 - 1} = 1 \rightarrow a^2 = 2 \rightarrow a = \pm \sqrt{2} \rightarrow \text{جواب: } \pm \sqrt{2}$$

$$f(x+y) = x^2 - \omega x + y \quad , \quad f(\varepsilon) \times f(-3) = ?$$

$$x=1 \rightarrow 1 - \omega + y = 1 \rightarrow f(\varepsilon) = 1$$

$$x=-3 \rightarrow 9 - 3\omega + y = 1 \rightarrow f(-3) = -2$$

$$\Rightarrow f(\varepsilon) \times f(-3) = 1 \times (-2) = -2$$

$$f(\cos^2 x + \sin^2 x) = \frac{1}{\sqrt{2}} \quad , \quad f\left(\frac{1}{\sqrt{2}}\right) = ?$$

$$\cos^2 x = t \rightarrow t^2 + \sin^2 t = \frac{1}{\sqrt{2}} \rightarrow t^2 + 1 - t^2 = \frac{1}{\sqrt{2}} \rightarrow (t-1)(t+1) = 0$$

$$\rightarrow t = 1 \quad , \quad t = -\frac{1}{\sqrt{2}} \quad \text{و } -1 < \cos x \leq 1 \rightarrow \cos x = \frac{1}{\sqrt{2}} \rightarrow \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \frac{1}{2}}{\frac{1}{2}} = 1$$

$$\rightarrow \tan^2 x = 1 \rightarrow f\left(\frac{1}{\sqrt{2}}\right) = \tan^2 x = 1 \rightarrow \text{جواب: } 1$$