

الف) $L(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \sqrt{\frac{x^2 - x + 1 - x^2}{x(x-1)}} \rightarrow \sqrt{\frac{1-x}{x(x-1)}}$ ریشه ها: $\frac{1}{x} = 0$ و $\frac{1}{x-1} = 0$ $\rightarrow \frac{0}{+\infty + \infty} = 0$ $\rightarrow$ جواب: $\emptyset$

ب) $\frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+1}} = \frac{\frac{x}{x+1} - \frac{x+1}{x}}{\frac{x}{x-1} + \frac{x+1}{x+1}} = \frac{\frac{x^2 - (x+1)(x+1)}{x(x+1)}}{\frac{x^2 + (x+1)(x-1)}{x(x-1)(x+1)}}$ $\rightarrow \frac{x^2 - (x+1)^2}{x^2 + (x+1)(x-1)}$ $\rightarrow \frac{x^2 - x^2 - 2x - 1}{x^2 + x^2 - 1}$ $\rightarrow \frac{-2x-1}{2x^2-1}$ $\rightarrow \frac{-2x-1}{2x^2-1} = 0 \rightarrow -2x-1=0 \rightarrow x = -\frac{1}{2}$ $\rightarrow$ جواب: $\{-\frac{1}{2}\}$

ج) $\frac{1}{x^2+1} = 0 \rightarrow x^2+1=0 \rightarrow x^2=-1 \rightarrow x = \pm i$ $\rightarrow$ جواب: $\{i, -i\}$

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الف) $f(x) = \left[\left(\left(\frac{1}{x} \right)^x - 9 \right) (32 - x^x) \right] \rightarrow \left(\left(\frac{1}{x} \right)^x - 9 \right) (32 - x^x) \geq 0 \rightarrow \left(\frac{1}{x} \right)^x - 9 \geq 0 \rightarrow x^{-x} \geq 3^2 \rightarrow -x \geq 2 \rightarrow$
 $x \leq -2$ و $32 - x^x \geq 0 \rightarrow x^x \leq 32 \rightarrow x \leq \frac{5}{2} \rightarrow \underline{D = (-\infty, -2] \cup \left[\frac{5}{2}, +\infty \right)}$ جواب.

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow x-1 \geq 0 \rightarrow x \geq 1$ و $\sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10$
 $\sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow y+1 = (3 - \sqrt{x-1})^2 \rightarrow y = (3 - \sqrt{x-1})^2 - 1 \rightarrow \underline{D = [1, 10]}$ جواب.

$$f(x) = \frac{\log_e (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow x^2 - x - 2 > 0 \rightarrow (x - 2)(x + 1) > 0 \quad \begin{array}{c} -1 \quad 2 \\ + \quad - \quad + \end{array}$$

$$\rightarrow D = (-\infty, -1) \cup (2, +\infty) \text{ (I)} \quad \text{و} \quad \sqrt{(x-1)(x+1)} + 1 \quad \begin{array}{c} -1 \quad 1 \\ + \quad - \quad + \end{array} \rightarrow (-\infty, -1] \cup [1, +\infty) \text{ (II)}$$

$$\rightarrow I \cap II = I \rightarrow \underline{D} = (-\infty, -1) \cup (2, +\infty) \rightarrow \text{جواب}$$

$$f(x) = \sqrt{1+ax-x^2} \Rightarrow D_f = [-1, b] \rightarrow 1+ax-x^2 \geq 0 \xrightarrow{\text{ریشه ها}} x = -1 \text{ و } x = b$$

$$f(x) = \begin{cases} 1-x & x \geq 1 \\ x & x < 1 \end{cases}, \quad g(x) = \sqrt{f(x)-x} \rightarrow f(x)-x \geq 0$$

$$\begin{aligned} & \rightarrow x \geq 1 \rightarrow 1-x-x \geq 0 \rightarrow 1-2x \geq 0 \rightarrow x \leq \frac{1}{2} \\ & \rightarrow x < 1 \rightarrow x-x \geq 0 \rightarrow 0 \geq 0 \rightarrow x \geq -1 \end{aligned}$$

$$\rightarrow D = [-1, \frac{1}{2}]$$

$$f(x) = x^2 - 1 \quad \sin \frac{x}{\sqrt{2}} = \frac{\sqrt{x}}{\sqrt{2}} \quad \cos 2x = 1$$

$$f(\sin \frac{x}{\sqrt{2}} + \cos 2x) = 1 \left(\frac{\sqrt{x}}{\sqrt{2}} + 1 \right)^2 + 1 \left(\frac{\sqrt{x}}{\sqrt{2}} + 1 \right) = 1 \left(\frac{x}{2} + 1 + \sqrt{x} \right) + \left(\sqrt{x} + 1 \right)$$

$$\frac{x}{2} + 1 - \varepsilon = \frac{x}{2} - 1 = \frac{-1}{2} \rightarrow \text{جواب}$$

$$f(x) = \frac{x+1}{x-1} \quad , \quad f(f(a)) = a$$

$$f(f(a)) = \frac{f(a)+1}{f(a)-1} \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a \rightarrow f(f(a)) = \frac{2a}{2} = a$$

$$\rightarrow a \in \mathbb{R} - \{1\} \rightarrow \text{جواب} \quad \text{①} \text{ ملاحظه کنید}$$

$$f(x) = x^2 - 1 \quad g(x) = \sqrt{x^2 - 1} \quad f(g(a)) = -\varepsilon \quad a = ?$$

$$g(a) = t \rightarrow t^2 - 1t = -\varepsilon \rightarrow t^2 - \varepsilon t + \varepsilon = 0 \rightarrow (t-1)^2 = 0$$

$$\rightarrow t = 1 \rightarrow \sqrt{a^2 - 1} = \varepsilon \rightarrow a^2 = \varepsilon^2 + 1 \rightarrow a = \pm \sqrt{\varepsilon^2 + 1} \rightarrow \text{جواب}$$

$$f(x+y) = x^2 - \omega x + y \quad , \quad f(\varepsilon) \times f(-3) = ?$$

$$x=1 \rightarrow 1 - \omega + y = 1 \rightarrow f(\varepsilon) = 1$$

$$x=-3 \rightarrow 9 - 3\omega + y = 1 \rightarrow f(-3) = -2$$

$$\Rightarrow f(\varepsilon) \times f(-3) = 1 \times (-2) = -2 \rightarrow \text{جواب}$$

$$f(\cos^2 x + \omega \cos x) = \frac{1}{\sqrt{2}} \quad , \quad f\left(\frac{1}{\sqrt{2}}\right) = ?$$

$$\cos x = t \rightarrow 1t^2 + \omega t = \frac{1}{\sqrt{2}} \rightarrow 1t^2 + \omega t - \frac{1}{\sqrt{2}} = 0 \rightarrow (t-1)(t+1) = 0$$

$$\rightarrow t = \frac{1}{\sqrt{2}} \quad , \quad t = -\frac{1}{\sqrt{2}} \quad \text{و چون } -1 < \cos x \leq 1 \rightarrow \cos x = \frac{1}{\sqrt{2}} \rightarrow \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \frac{1}{2}}{\frac{1}{2}} = 1 \rightarrow \tan x = \pm 1$$

$$\rightarrow \tan^2 x = \frac{1 - \frac{1}{2}}{\frac{1}{2}} = 1 \rightarrow f\left(\frac{1}{\sqrt{2}}\right) = \tan^2 x \rightarrow f\left(\frac{1}{\sqrt{2}}\right) = 1 \rightarrow \text{جواب}$$