

$$(الف) \frac{n-1}{n} - \frac{n}{n-1} = \frac{n^2 - n + 1 - n^2}{n^2 - n} \Rightarrow \frac{-n+1}{n(n-1)} \Rightarrow \frac{-(n-1)}{n(n-1)} > 0$$

$$\Rightarrow \frac{0}{+} \frac{+}{-} \frac{+}{+} \rightarrow (-\infty, 1) \cup (1, +\infty)$$

$$(ب) \frac{c}{n-1} + \frac{1}{n+c} \neq 0 \Rightarrow \frac{c(n+c) + (n-1)}{(n-1)(n+c)} \neq 0 \Rightarrow cn + c^2 + n - 1 \neq 0 \Rightarrow c^2 + c + n - 1 \neq 0$$

$$\Rightarrow n \neq \frac{1-c}{c} \rightarrow \{R - \{-\frac{1}{c}, -\frac{1}{c+1}, 0, 1\}\}$$

$$(الف) (\frac{1}{x})^n - 9 = 0 \rightarrow (\frac{1}{x})^n = 9 \rightarrow \frac{1}{x^n} = 9 \rightarrow x^n = \frac{1}{9} \rightarrow x = \pm \frac{1}{3}$$

$$cn - \frac{1}{x} = 0 \rightarrow cn = \frac{1}{x} \rightarrow x = \frac{1}{cn} \rightarrow \frac{-1}{cn} \rightarrow (-\infty, -\frac{1}{cn}] \cup [\frac{1}{cn}, +\infty)$$

$$(ب) x-1 > 0 \rightarrow x > 1, y+1 > 0 \rightarrow y > -1 \Rightarrow \sqrt{y+1} = \sqrt{x-1}$$

$$\Rightarrow \sqrt{x-1} > 0 \Rightarrow x > 1 \Rightarrow \sqrt{x-1} \geq 1 \Rightarrow x-1 \geq 1 \Rightarrow x \geq 2 \Rightarrow [2, +\infty)$$

$$I \quad n^2 - n - 2 > 0 \rightarrow (n-2)(n+1) > 0 \rightarrow \frac{-1}{n-2} \rightarrow (-\infty, -1) \cup (2, +\infty)$$

$$II \quad n-1 > 0 \rightarrow (n-1)(n+1) > 0 \rightarrow \frac{-1}{n-1} \rightarrow (-\infty, -1) \cup (1, +\infty)$$

$$III \quad \sqrt{n-1} + 1 > 0 \rightarrow I \cap II \cap III \rightarrow (-\infty, -1) \cup (2, +\infty)$$

به ازای هر n که II
محقق کننده شود
I و III

$$cn - 2a - \frac{1}{c} = 0 \rightarrow a = \frac{cn-1}{2c} \rightarrow a = \frac{1}{2c} \Rightarrow cn - \frac{1}{c} - b^2 = 0 \Rightarrow cn^2 + b^2 - \frac{1}{c} = 0$$

$$\Rightarrow b^2 + b - \frac{1}{c} = 0 \rightarrow (b+\frac{1}{2})^2 - \frac{1}{4} - \frac{1}{c} = 0 \Rightarrow (b+\frac{1}{2})^2 = \frac{1}{4} + \frac{1}{c} \Rightarrow b+\frac{1}{2} = \pm \sqrt{\frac{1}{4} + \frac{1}{c}} \Rightarrow b = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{1}{c}}$$

$$f(n) - n > 0 \rightarrow f(n) > n \quad \begin{cases} cn - c > n \rightarrow cn > n+c \rightarrow n > 1 \\ cn + c > n \rightarrow n > -c \end{cases} \rightarrow n \rightarrow [2, +\infty)$$

$$\sin \frac{\pi}{2} = \frac{\sqrt{2}}{2}, \cos \frac{\pi}{2} = 1 \rightarrow f\left(1 + \frac{\sqrt{2}}{2}\right) \Rightarrow 2\left(1 + \frac{\sqrt{2}}{2}\right)^2 - 2\left(1 + \frac{\sqrt{2}}{2}\right) - 9$$

$$= 2 + \frac{\sqrt{2}}{2} + 2\sqrt{2} - 2 - 2\sqrt{2} = \frac{\sqrt{2}}{2} - 2 = -\frac{1}{2}$$

$$f(a) = \frac{a+1}{a-1} \rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} \Rightarrow \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a \rightarrow \mathbb{R} - \{1\}$$

$$\sqrt{a^2-1} \rightarrow \frac{(\sqrt{a^2-1})^2}{t^2} - \frac{2\sqrt{a^2-1}}{t} = 0 \rightarrow t^2 - 2t + 1 = 0 \rightarrow (t-1)^2 = 0$$

$$\Rightarrow t=1 \rightarrow \sqrt{a^2-1} = 1 \rightarrow a^2-1 = 1 \rightarrow a^2 = 2 \rightarrow a = \pm\sqrt{2}$$

$$\begin{aligned} x+y &= -3 \rightarrow x = -4 \Rightarrow 2x+3y+4=12 \\ x+y &= 2 \rightarrow x = 1 \rightarrow 1-2+4=2 \end{aligned} \left. \vphantom{\begin{aligned} x+y &= -3 \\ x+y &= 2 \end{aligned}} \right\} \forall x, y = 1, 2$$

$$c \cos \theta + d \cos \theta = \frac{1}{d} \rightarrow c \cos \theta + d \cos \theta - \frac{1}{d} = 0 \rightarrow c \cos \theta + d \cos \theta - \frac{1}{d} = 0$$

$$\Rightarrow c \cos \theta = \frac{1}{d} + \frac{1}{c} \Rightarrow \left(c \times \left(\frac{1}{c} \right) + d \times \left(\frac{1}{d} \right) \right) = \frac{1}{d}$$

$$\Rightarrow x = \frac{1}{c} \rightarrow f\left(\frac{1}{c}, d\right) = \tan^{-1} \frac{1}{c} \rightarrow f\left(\frac{1}{c}, d\right) = \left(\frac{1}{c} \right)^2 = \frac{1}{c^2}$$