

(الف) $\frac{n-1}{n} - \frac{n}{n-1} = \frac{n^2 - n + 1 - n^2}{n^2 - n} \Rightarrow \frac{-n+1}{n(n-1)} \Rightarrow \frac{-n+1}{n(n-1)} > 0$

$\Rightarrow \frac{0}{+} \frac{+}{-} \frac{+}{+} \rightarrow (-\infty, 0) \cup (\frac{1}{n}, +\infty)$

(ب) $\frac{c}{n-1} + \frac{1}{n+c} \neq 0 \Rightarrow \frac{c(n+c) + (n-1)}{(n-1)(n+c)} \neq 0 \Rightarrow cn + c^2 + n - 1 \neq 0 \Rightarrow c^2 + n + c - 1 \neq 0$

$\Rightarrow n \neq \frac{-c}{c^2} \rightarrow \{R - \{-\frac{1}{c}, -\frac{2}{c}, -1, 0, 1\}\}$

(الف) $(\frac{1}{x})^n - 9 = 0 \rightarrow (\frac{1}{x})^n = 9 \rightarrow \frac{1}{x^n} = 9 \rightarrow x^n = \frac{1}{9} \rightarrow x = \pm \frac{1}{3}$

$cn - \frac{1}{x} = 0 \rightarrow cn = \frac{1}{x} \rightarrow x = \frac{1}{cn} \rightarrow \frac{-1}{cn} \rightarrow (-\infty, -\frac{1}{cn}] \cup [\frac{1}{cn}, +\infty)$

(ب) $x-1 > 0 \rightarrow x > 1$, $y+1 > 0 \rightarrow y > -1$ $\Rightarrow \sqrt{y+1} = \sqrt{x-1}$

$\Rightarrow \sqrt{x-1} > 0 \Rightarrow x > 1$ $\Rightarrow \sqrt{x-1} > 1 \Rightarrow x-1 > 1 \Rightarrow x > 2 \Rightarrow [2, +\infty)$

I $n^2 - n - 2 > 0 \rightarrow (n-2)(n+1) > 0 \rightarrow \frac{-1}{n-2} \rightarrow (-\infty, -1) \cup (2, +\infty)$

II $n-1 > 0 \rightarrow (n-1)(n+1) > 0 \rightarrow \frac{-1}{n-1} \rightarrow (-\infty, -1) \cup (1, +\infty)$

III $\sqrt{n-1} + 1 > 0 \rightarrow I \cap II \cap III \rightarrow (-\infty, -1) \cup (2, +\infty)$

به ازای هر n که II
محقق شود
I و III

$cn - 2a - \frac{1}{c} = 0 \rightarrow 1 = 2a \rightarrow a = \frac{1}{2} \Rightarrow cn - \frac{b}{c} - b^2 = 0 \Rightarrow cb^2 + b - \frac{cn}{c} = 0$

$\Rightarrow b^2 + b - \frac{cn}{c} = 0 \rightarrow (b+\frac{1}{2})(b-\frac{cn}{c}) = 0 \Rightarrow b = -\frac{1}{2}, \frac{cn}{c} \Rightarrow a+b = -\frac{1}{2} + \frac{cn}{c} = 1$

$f(n) - n > 0 \rightarrow f(n) > n$ $cn - c > n \rightarrow cn > n + c \rightarrow n > 1$
 $cn + c > n \rightarrow n > cn - c$ $\Rightarrow n \rightarrow [2, +\infty)$

$$\sin \frac{\pi}{2} = \frac{\sqrt{2}}{2}, \cos \frac{\pi}{2} = 1 \rightarrow f\left(1 + \frac{\sqrt{2}}{2}\right) \Rightarrow 2\left(1 + \frac{\sqrt{2}}{2}\right)^2 - 2\left(1 + \frac{\sqrt{2}}{2}\right) \quad -9$$

$$= 2 + \frac{2}{2} + 2\sqrt{2} - 2 - 2\sqrt{2} = \frac{2}{2} - 2 = -\frac{1}{2}$$

$$f(a) = \frac{a+1}{a-1} \rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} \Rightarrow \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a \rightarrow \mathbb{R} - \{1\} \quad -\checkmark$$

$$\sqrt{a^2-1} \rightarrow \frac{(\sqrt{a^2-1})^2}{t^2} - \frac{2\sqrt{a^2-1}}{t} = 0 \rightarrow t^2 - 2t + 1 = 0 \rightarrow (t-1)^2 = 0$$

$$\Rightarrow t=1 \rightarrow \sqrt{a^2-1} = 1 \rightarrow a^2-1 = 1 \rightarrow a^2 = 2 \rightarrow a = \pm\sqrt{2}$$

$$\begin{aligned} u+v &= -1 \rightarrow u = -1 \Rightarrow 29 + 10 + 9 = 48 \\ u+v &= 1 \rightarrow u = 1 \rightarrow 1 - 2 + 9 = 8 \end{aligned} \quad \left. \vphantom{\begin{aligned} u+v &= -1 \\ u+v &= 1 \end{aligned}} \right\} \forall x \in \mathbb{Z} \quad -9$$

$$c \cos u + d \cos u = \frac{1}{2} \rightarrow c \cos u + d \cos u - \frac{1}{2} = 0 \rightarrow c \cos u + d \cos u - \frac{1}{2} = 0$$

$$\Rightarrow c \cos u = \frac{1}{2} + \frac{1}{2} \Rightarrow \left(\cos\left(\frac{1}{2}\right) + \cos\left(\frac{1}{2}\right) \right) = \frac{1}{2}$$

$$\Rightarrow u = \frac{\pi}{2} \rightarrow f\left(\frac{\pi}{2}\right) = \tan \frac{\pi}{2} \rightarrow f\left(\frac{\pi}{2}\right) = (\sqrt{2})^2 = 2$$