

الف) $f(n) = \sqrt{\frac{n-1}{n} - \frac{n}{n-1}}$ $\Rightarrow \frac{n-1}{n} - \frac{n}{n-1} \geq 0 \Rightarrow \frac{n^2 - 2n + 1 - n^2}{n(n-1)} \geq 0$ $\frac{-2n+1}{n(n-1)} \geq 0$ $\frac{1}{n-1} \geq \frac{1}{n}$ $D_f = (-\infty, 0) \cup [\frac{1}{2}, +\infty)$ (1)

ب) $f(n) = \frac{\frac{1}{n+1} - \frac{2}{n}}{\frac{2}{n-1} + \frac{1}{n+2}}$ $\Rightarrow n \neq 0, -1, 1, -2$ $\frac{2}{n-1} + \frac{1}{n+2} \neq 0 \Rightarrow \frac{2n+2}{n^2+n-2} \neq 0 \Rightarrow D_f = \mathbb{R} \setminus \{-2, -\frac{2}{3}, \frac{1}{2}, 1\}$ (2)

الف) $f(n) = \sqrt{((\frac{1}{n})^2 - 9)(12 - 4^n)}$ $\frac{-2}{n-1} \geq \frac{\frac{9}{n^2}}{n}$ $D_f = (-\infty, -2] \cup [\frac{9}{2}, +\infty)$ (3)

ب) $\sqrt{x-1} + \sqrt{y+1} = 3$ $\sqrt{x-1} \geq 0$ $\sqrt{y+1} \geq 0$ $\sqrt{x-1} \leq 3$ $1 \leq x-1 \leq 9$ $2 \leq x \leq 10$ $D_f = [2, 10]$ (4)

$f(n) = \frac{\log_e (n^2 - n - 2)}{\sqrt{n^2 - 1} + 1}$ $n^2 \geq 1 \Rightarrow n \geq 1, n \leq -1$ $n^2 - n - 2 > 0 \Rightarrow (n+1)(n-2) > 0$ $\frac{-1}{n-1} \geq \frac{2}{n}$ $n > 2$ $n < -1$ $D_f = (-\infty, -1) \cup (2, +\infty)$ (5)

$f(n) = \sqrt{a + an - n^2}$ $D_f = [-2, b]$ $D_f = -2 = -2 \times b \Rightarrow b = \frac{2}{-2} = -1$ $S = a = \frac{2}{-2} + (-2) = -1$ (6)

$f(n) = \begin{cases} 2n-2 & n \geq 1 \\ 2n+3 & n < 1 \end{cases}$ $g(n) = \sqrt{f(n) - n}$ $f(n) \geq n$ $2n-2 \geq n \Rightarrow n \geq 2$ $2n+3 \geq n \Rightarrow n \geq -3$ $D_f = [-3, +\infty)$ (7)

$f(n) = 2n^2 - 2n$ $f(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}) = f(1 + \frac{\sqrt{2}}{2}) = 2n(n-2) = (\frac{\sqrt{2}}{2} - 1)(\frac{\sqrt{2}}{2} - 1) = (\frac{\sqrt{2}}{2} - 1)^2 = \frac{1}{2} - \sqrt{2} + 1 = \frac{3}{2} - \sqrt{2}$ (8)

$f(n) = \frac{n+1}{n-1}$ $f(f(a)) = a \Rightarrow f(\frac{a+1}{a-1}) = a \Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \Rightarrow \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a$ (9)

$f(n) = n^2 - 2n$ $g(n) = \sqrt{n^2 - 1}$ $f(g(a)) = -2$ $\sqrt{a^2 - 1} = 2 \Rightarrow a^2 = 5 \Rightarrow a = \pm \sqrt{5}$ (10)

$$f(n+r) = n^r - an + q$$

$$n = -q$$

$$f(-q) = -q \times -a = va$$

$$f(n+r) = (n-r)(n-r)$$

$$f(\varepsilon) = -r \times -1 = r$$

$$\left. \begin{array}{l} f(-q) = f(\varepsilon) = 1 \end{array} \right\} f(-q) = f(\varepsilon) = 1$$

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$$\{ \cos^r n + a \cos n = r/a \Rightarrow \cos^r n + a(\cos n - 1) \geq 0 \Rightarrow (\cos n + 1)(\cos n - 1) = 0 \Rightarrow \cos n = -\frac{1}{\varepsilon}, \frac{1}{\varepsilon} = \frac{1}{r}$$

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$$f(r/a) = \tan^r n = \frac{1}{\cos^r n} - 1 = r$$

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