

نام و نام خانوادگی: پاسخنامه تشریحی تکلیف شماره ۵۰ کلاس B - ۳۰۰

۲. اولین معادله نوشت

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \Rightarrow \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$
 ۱ و ۰: ریشه مرتبه ۱ و ۲: ریشه مرتبه ۲
 $\frac{0}{+} - \frac{1}{+} + \frac{1}{+} = \Rightarrow ID_f = (-\infty, 0) \cup [\frac{1}{4}, +\infty)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+1}} = \frac{\frac{x - (x+1)}{x(x+1)}}{\frac{x-1 + x+1}{(x-1)(x+1)}} = \frac{(-x-1)(x-1)(x+1)}{(x^2-1)(x+1)x} \Rightarrow \frac{-x-1}{x(x-1)}$
 $\frac{0}{+} - \frac{1}{+} + \frac{1}{+} = \Rightarrow ID_f = \mathbb{R} - \{-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}\}$

الف) $f(x) = \sqrt{(\frac{1}{x}-4)(3x-x^2)} \geq 0 \Rightarrow (\frac{1}{x}-4) \geq 0 \Rightarrow (\frac{1}{x})^2 - 4 = 0 \Rightarrow (\frac{1}{x})^2 = 4 \Rightarrow -x = 2 \Rightarrow x = -2$
 $\frac{-2}{+} - \frac{4}{+} + \frac{1}{+} = \Rightarrow ID_f = (-\infty, -2] \cup [\frac{1}{4}, +\infty)$

ب) $\sqrt{y+1} + \sqrt{y-1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{y-1}$
 $ID_f = [1, 10]$

$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$
 I) $x^2 - x - 2 > 0 \Rightarrow x < -1$ یا $x > 2$
 II) $\sqrt{x^2 - 1} \geq 0 \Rightarrow x^2 - 1 \geq 0 \Rightarrow x \leq -1$ یا $x \geq 1$
 III) $\sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1$
 $ID_f = (-\infty, -1) \cup (2, +\infty)$

$-2 + b = -2 + b \Rightarrow -(n+2)(n-b) = -n^2 + an + 3 \Rightarrow a = -2 + b$
 $(n+2)(n-b) = n^2 - an - 3 \Rightarrow -a = -5 \Rightarrow a = 5$
 $a + b = -\frac{1}{4} + \frac{3}{4} = 1 \Rightarrow a + b = 1$

$g(x) = \begin{cases} \sqrt{\frac{3x-2-x}{f(x)}} & x \geq 1 \\ \sqrt{\frac{2x+3-x}{f(x)}} & x < 1 \end{cases} \Rightarrow g(x) = \begin{cases} \sqrt{2x-2} & x \geq 1 \rightarrow [1, +\infty) \\ \sqrt{x+3} & x < 1 \rightarrow [-3, 1) \end{cases}$

$ID_g = [-3, +\infty)$

$$\underbrace{F\left(\sin \frac{\pi}{4} + \cos 2\pi\right)}_{\frac{\sqrt{2}}{2}} = F\left(\frac{\sqrt{2}}{2} + 1\right) \Rightarrow F\left(\frac{\sqrt{2}}{2} + 1\right) = 2\left(\frac{\sqrt{2}}{2} + 1\right)^2 - 4\left(\frac{\sqrt{2}}{2} + 1\right) \Rightarrow$$

$$F(x) = 2x^2 - 4x$$

$$F\left(\frac{\sqrt{2}}{2} + 1\right) = 2\left(\frac{\sqrt{2}}{2} + 1 + \sqrt{2}\right) - 4\left(\frac{\sqrt{2}}{2} + 1\right) = \frac{\sqrt{2}}{2} + 2 + 2\sqrt{2} - 2\sqrt{2} - 4 = -\frac{1}{2} \Rightarrow$$

$$F\left(\sin \frac{\pi}{4} + \cos 2\pi\right) = -\frac{1}{2}$$

$$F(x) = \frac{x+1}{x-1} \Rightarrow F(a) = \frac{a+1}{a-1} \Rightarrow F(F(a)) = F\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} \Rightarrow$$

$$F(F(a)) = \frac{\frac{(a+1) + (a-1)}{(a-1)}}{\frac{(a+1) - (a-1)}{(a-1)}} = \frac{2a}{2} = a \text{ و } F(F(a)) - a = f\left(\frac{a+1}{a-1}\right) - a = a - a = 0$$

ولی ما در محاسبه $F(F(a))$ در مرحله‌ی بعد از خروج مشترک گرفتن، خروجی $2a$ را به صورت $2(a-1)$ (یعنی $a-1$) با هم ساده کردیم پس ما باید در صورتی که $a-1$ را از $2a$ ساده کنیم، خروجی نباید صفر شود $\Leftarrow$
 $a-1 \neq 0 \Rightarrow a \neq 1$ و $(a+1) - (a-1) \neq 0 \Rightarrow 2 \neq 0$ $\Rightarrow$ $D_F = \mathbb{R} - \{1\}$

$$g(a) = z \Rightarrow F(z) = z^2 - 4z = -4 \Rightarrow z^2 - 4z + 4 = 0 \xrightarrow[\Delta = 0]{\substack{a = 0 \\ b = -4}} z = 2 \Rightarrow g(a) = 2 \Rightarrow$$

$$\left. \begin{array}{l} g(x) = \sqrt{x^2 - 1} \\ g(a) = 2 \end{array} \right\} \rightarrow \sqrt{a^2 - 1} = 2 \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow a = \pm \sqrt{5}$$

$$F(x+3) = x^2 - 5x + 4 = (x-2)(x-3) \Rightarrow F(x) = (x-5)(x-4) \Rightarrow$$

$$F(4) = (4-5)(4-4) = -1 \times 0 = 0$$

$$F(-3) = (-3-5)(-3-4) = -8 \times -9 = 72 \Rightarrow F(4) \times F(-3) = 0 \times 72 = 0 \Rightarrow$$

$$F(4) \times F(-3) = 0$$

$$F(\sqrt{3}\cos x + \cos x) = \tan^2 x \quad \left(\text{ما فرض می‌کنیم که } F(\sqrt{3}\cos x + \cos x) \text{ و } F(\frac{1}{\sqrt{3}}) \text{ با هم برابرند} \right)$$

$$F\left(\frac{1}{\sqrt{3}}\right) = F(\sqrt{3}\cos x + \cos x) \Rightarrow \sqrt{3}\cos^2 x + \cos x - \frac{1}{\sqrt{3}} = 0 \Rightarrow$$

$$\cos x = t \Rightarrow \sqrt{3}t^2 + \cos x - \frac{1}{\sqrt{3}} = 0 \Rightarrow \sqrt{3}t^2 + \cos x - 1 = 0 \Rightarrow (t-2)(t+1) = 0 \Rightarrow \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ و } \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow$$

$$\cos x = \frac{1}{\sqrt{3}} \text{ و } \left(\frac{-1}{\sqrt{3}}\right) \rightarrow \frac{1}{\sqrt{3}} = -\frac{1}{\sqrt{3}} \Rightarrow \cos x = \frac{1}{\sqrt{3}} \Rightarrow \{x \mid x = 2k\pi \pm \frac{\pi}{\sqrt{3}} \text{ و } k \in \mathbb{Z}\}$$

$$\Rightarrow \tan^2 x = (\sqrt{3})^2 = 3 \Rightarrow F(\sqrt{3}) = 3$$