

الف)  $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0 \Rightarrow \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$   
 $\Rightarrow \frac{(x-1-x)(x-1+x)}{x(x-1)} \geq 0 \Rightarrow \frac{-1}{x} \geq 0 \Rightarrow x < 0$

$\Rightarrow ID_f = (-\infty, 0) \cup [\frac{1}{x}, 1)$

ب)  $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+1}} \geq \frac{x - 2(x+1)}{x(x+1)} = \frac{-x-2}{x(x+1)} \geq 0 \Rightarrow \frac{-x-2}{x(x+1)} \geq 0$   
 $\Rightarrow \frac{-x-2}{x(x+1)} \geq 0 \Rightarrow \frac{-x-2}{x(x+1)} \geq 0 \Rightarrow \frac{-x-2}{x(x+1)} \geq 0$

$ID_f = \mathbb{R} - \{-\frac{2}{x} - \frac{1}{x+1}\}$

الف)  $f(x) = \sqrt{(\frac{1}{x}-1)(3x-1)} \geq 0 \Rightarrow (\frac{1}{x}-1)(3x-1) \geq 0 \Rightarrow \frac{1}{x} - 1 \geq 0 \Rightarrow \frac{1}{x} \geq 1 \Rightarrow x \leq 1$   
 $\Rightarrow \frac{1}{x} - 1 \geq 0 \Rightarrow \frac{1}{x} \geq 1 \Rightarrow x \leq 1$

$\Rightarrow ID_f = (-\infty, -1] \cup [\frac{1}{x}, +\infty)$

ب)  $\sqrt{y+1} + \sqrt{y-1} = 2 \Rightarrow \sqrt{y+1} = 2 - \sqrt{y-1}$   
 $\Rightarrow \sqrt{y+1} = 2 - \sqrt{y-1} \Rightarrow \sqrt{y+1} = 2 - \sqrt{y-1} \Rightarrow \sqrt{y+1} = 2 - \sqrt{y-1}$

$ID_f = [1, 2]$

I)  $x^2 - x - 2 > 0 \Rightarrow x^2 - x - 2 > 0 \Rightarrow x^2 - x - 2 > 0 \Rightarrow x^2 - x - 2 > 0$   
 $\Rightarrow x^2 - x - 2 > 0 \Rightarrow x^2 - x - 2 > 0 \Rightarrow x^2 - x - 2 > 0$

III)  $\sqrt{x^2-1} + 1 \neq 0 \Rightarrow \sqrt{x^2-1} \neq -1$

$ID_f = (-\infty, -1) \cup (1, +\infty)$

$a + b = -\frac{1}{x} + \frac{3}{x} = 1 \Rightarrow a + b = 1$

$g(x) = \begin{cases} \sqrt{\frac{3x-2-x}{x}} & x \geq 1 \\ \sqrt{\frac{2x+3-x}{x}} & x < 1 \end{cases} \Rightarrow g(x) = \begin{cases} \sqrt{2x-2} & x \geq 1 \\ \sqrt{x+3} & x < 1 \end{cases}$

$ID_g = [-2, +\infty)$

$$\left. \begin{aligned} F\left(\sin \frac{\pi}{4} + \cos 2\pi\right) &= F\left(\frac{\sqrt{2}}{2} + 1\right) \\ F(x) &= 2x^2 - 4x \end{aligned} \right\} \Rightarrow F\left(\frac{\sqrt{2}}{2} + 1\right) = 2\left(\frac{\sqrt{2}}{2} + 1\right)^2 - 4\left(\frac{\sqrt{2}}{2} + 1\right) \Rightarrow$$

$$F\left(\frac{\sqrt{2}}{2} + 1\right) = 2\left(\frac{2}{2} + 1 + \sqrt{2}\right) - 4\left(\frac{\sqrt{2}}{2} + 1\right) = 2 + 2 + 2\sqrt{2} - 2\sqrt{2} - 4 = -1 \Rightarrow$$

$$\boxed{F\left(\sin \frac{\pi}{4} + \cos 2\pi\right) = -\frac{1}{2}}$$

$$F(x) = \frac{x+1}{x-1} \Rightarrow F(a) = \frac{a+1}{a-1} \Rightarrow F(F(a)) = F\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} \Rightarrow$$

$$F(F(a)) = \frac{\frac{(a+1) + (a-1)}{(a-1)}}{\frac{(a+1) - (a-1)}{(a-1)}} = \frac{2a}{2} = a \text{ و } F(F(a)) - a = f\left(\frac{a+1}{a-1}\right) - a = a - a = 0$$

ولی ما در محاسبه  $F(F(a))$  در مرحله‌ی بعد از خروج مشترک کسرها، خروجی  $2a$  کسر صحت و خروجی  $(a-1)$  را با هم ساده کردیم پس ما باید دقت را در قبل و از انجام این عمل ما سبک کنیم چون خروجی نباید صفر شود  $\Rightarrow$   $a-1 \neq 0 \Rightarrow a \neq 1$  و  $(a+1) - (a-1) \neq 0 \Rightarrow 2 \neq 0$   $\Rightarrow$   $ID_F = IR - \{1\}$

$$g(a) = z \Rightarrow F(z) = z^2 - 4z = -4 \Rightarrow z^2 - 4z + 4 = 0 \xrightarrow[\Delta = 0]{\frac{-b}{2a} = 2} z = 2 \Rightarrow g(a) = 2 \Rightarrow$$

$$\left. \begin{aligned} g(x) &= \sqrt{x^2 - 1} \\ g(a) &= 2 \end{aligned} \right\} \rightarrow \sqrt{a^2 - 1} = 2 \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow \boxed{a = \pm \sqrt{5}}$$

$$F(x+3) = x^2 - 5x + 4 = (x-2)(x-3) \Rightarrow F(x) = (x-5)(x-4) \Rightarrow$$

$$\left. \begin{aligned} F(4) &= (4-5)(4-4) = -1 \times 0 = 0 \\ F(-3) &= (-3-5)(-3-4) = -8 \times -7 = 56 \end{aligned} \right\} \Rightarrow F(4) \times F(-3) = 0 \times 56 = 0 \Rightarrow$$

$$\boxed{F(4) \times F(-3) = 0}$$

$$F(2\cos^2 x + 5\cos x) = \tan^2 x \quad \left( \text{ما فرض می‌کنیم که } F\left(\frac{1}{2}\right) = F(2\cos^2 x + 5\cos x) \text{ و } F\left(\frac{1}{2}\right) = \tan^2 x \right)$$

$$F\left(\frac{1}{2}\right) = F\left(\frac{1}{2}\right)$$

$$\rightarrow F\left(\frac{1}{2}\right) = F(2\cos^2 x + 5\cos x) \Rightarrow 2\cos^2 x + 5\cos x - \frac{1}{2} = 0 \Rightarrow$$

$$\cos x = t \Rightarrow 2t^2 + 5t - \frac{1}{2} = 0 \Rightarrow 4t^2 + 10t - 1 = 0 \Rightarrow (t-2)(t+7) = 0 \Rightarrow \frac{1}{2} = \frac{1}{2} \text{ و } \frac{1}{2} = \frac{1}{2} \Rightarrow$$

$$\cos x = \frac{1}{2} \text{ و } \left(\frac{-5}{4}\right) \rightarrow \frac{1}{2} = -\frac{1}{2} \vee \frac{1}{2} \Rightarrow \cos x = \frac{1}{2} \Rightarrow \{x \mid x = 2k\pi \pm \frac{\pi}{3} \text{ و } k \in \mathbb{Z}\}$$

$$\Rightarrow \tan^2 x = (\sqrt{3})^2 = 3 \Rightarrow \boxed{F\left(\frac{1}{2}\right) = 3}$$