

۱۹/۷۵ آزمون

الف) $\frac{x-1}{x} - \frac{x}{x-1} > 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} > 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} > 0 \rightarrow \frac{-2x + 1}{x(x-1)} > 0$
 $D_f = (-\infty, \frac{1}{2}) \cup (1, +\infty)$

ب) $x \neq -1, 0, 1, -2$
 $\frac{x}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{x(x+2) + (x-1)}{(x-1)(x+2)} \neq 0 \rightarrow \frac{x^2 + 3x - 1}{(x-1)(x+2)} \neq 0$
 $D_f = \mathbb{R} - \{-1, -\frac{1}{3}, 0, 1, 2\}$

الف) $(\frac{1}{x})^n - 9 > 0 \rightarrow (\frac{1}{x})^n > 9 \rightarrow \frac{1}{x^n} > 9 \rightarrow x^n < \frac{1}{9}$
 $D_f = \mathbb{R} - (-2, 2)$

ب) $\sqrt{x+1} = \sqrt{x} - \sqrt{x-1} \rightarrow \sqrt{x} - \sqrt{x-1} > 0 \rightarrow \sqrt{x} > \sqrt{x-1} \rightarrow x > x-1 \rightarrow x > 1$
 $D_f = [1, +\infty)$

$x^2 - x - 2 > 0 \rightarrow (x+2)(x-1) > 0 \rightarrow x < -2 \text{ or } x > 1$
 $x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow x > 1 \text{ or } x < -1$
 $\sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1$
 $D_f = (-\infty, -1) \cup (1, +\infty)$

$f(x) = \sqrt{x^2 + a} - x^2$
 $D_f = [-2, b]$
 $f(-2) = 0 \rightarrow 4 - 2a - 4 = 0 \rightarrow -2a = 0 \rightarrow a = 0$
 $a + b = ?$
 $a = -\frac{1}{4} \rightarrow f(x) = \sqrt{x^2 - \frac{1}{4}} - x^2$
 $x^2 + \frac{1}{4}x - 2 = 0 \rightarrow \Delta = \frac{1}{16} - 4(-2) = \frac{1}{16} + 8 = \frac{129}{16}$
 $a + b = -\frac{1}{4} + \frac{1}{4} = 0$

$g(x) = \sqrt{f(x)} - x$
 $f(x) - x > 0 \rightarrow f(x) > x \rightarrow x^2 - x^2 > x \rightarrow 0 > x \rightarrow x < 0$
 $x < 0 \rightarrow x^2 > 0 \rightarrow x^2 + \frac{1}{4}x - 2 > 0 \rightarrow x^2 + \frac{1}{4}x - 2 > 0$
 $D_f = [-2, +\infty)$

$$f(x) = 2x^2 - 1 \rightarrow f(2-1)$$

$$f\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) \rightarrow f\left(\frac{\sqrt{2}}{2} + 1\right) \rightarrow f\left(\frac{\sqrt{2}+2}{2}\right) \rightarrow \sqrt{2}+2\left(\frac{\sqrt{2}+2}{2}-1\right) \rightarrow \sqrt{2}+2\left(\frac{\sqrt{2}-2}{2}\right) = \frac{\sqrt{2}-2}{2} = -\frac{1}{2}$$

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$$f(x) = \frac{x+1}{x-1} \rightarrow f(a) = \frac{a+1}{a-1} \rightarrow f\left(\frac{a+1}{a-1}\right) = a \rightarrow \frac{\frac{a+1}{a-1}+1}{\frac{a+1}{a-1}-1} = a \rightarrow \frac{a+1+a-1}{a+1-a+1} = a \rightarrow \frac{2a}{2} = a$$

اگر $a=1$ باشد صفر عبارت منفر شود
د عبارت تعریف نشده شود!

$a \in \mathbb{R}$

رابطه تکرار برقرار است.

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$$a \in \mathbb{R} - \{1\} \rightarrow D_f = \mathbb{R} - \{1\}$$

1, 1/5

$$f(g(a)) = -1 \rightarrow f(a) = -1 \rightarrow a^2 - 1 = -1 \rightarrow a^2 = 0 \rightarrow a = 0$$

$$f(x) = x^2 - 1$$

$$g(x) = \sqrt{x^2 - 1}$$

$a = \pm \sqrt{1}$

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$$f(x+2) = x^2 - 2x + 4$$

$$f(4), f(-2)$$

$$f(4) = 4^2 - 2 \cdot 4 + 4 = 16 - 8 + 4 = 12$$

$$f(-2) = (-2)^2 - 2 \cdot (-2) + 4 = 4 + 4 + 4 = 12$$

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$$f(\cos^2 x + \sin^2 x) = \tan^2 x$$

$$f(1) = ?$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x + \sin^2 x - \frac{1}{2} = 0$$

$$\cos^2 x = \frac{1}{2}$$

$$\frac{-1 \pm \sqrt{1 - 4 \cdot (-\frac{1}{2})}}{2 \cdot (-\frac{1}{2})} = \frac{-1 \pm \sqrt{3}}{-1} = 1 \pm \sqrt{3}$$

$$\frac{1}{2} \sqrt{3}$$

$$f(1) = 3$$

$$\tan^2 x = 3 \rightarrow \tan x = \sqrt{3} \rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$\cos x = \frac{1}{2}$$

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$$-1 \leq \cos x \leq 1$$

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$$f(f(a)) = \frac{f(a)+1}{f(a)-1} = a \rightarrow \frac{\frac{a+1}{a-1}+1}{\frac{a+1}{a-1}-1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a \quad -V$$

$$x \in D_f \rightarrow x \in \mathbb{R} - \{1\}$$

پس به ازای تمامی نقاط در دامنه f برقرار است ✓