

الف) $\frac{x-1}{x} - \frac{x}{x-1} > 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} > 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} > 0 \rightarrow \frac{-2x + 1}{x(x-1)} > 0$
 $\frac{-2x + 1}{x(x-1)} > 0 \rightarrow \frac{1-2x}{x(x-1)} > 0$
 $D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $x \neq -1, 0, 1, -2$
 $\frac{x}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{x(x+2) + (x-1)}{(x-1)(x+2)} \neq 0 \rightarrow \frac{x^2 + 3x - 1}{(x-1)(x+2)} \neq 0$
 $\frac{x^2 + 3x - 1}{(x-1)(x+2)} \neq 0 \rightarrow x^2 + 3x - 1 \neq 0$
 $D_f = \mathbb{R} - \{-2, -\frac{1}{3}, -1, 0, 1\}$

الف) $(\frac{1}{x})^n - 9 > 0 \rightarrow (\frac{1}{x})^n > 9 \rightarrow \frac{1}{x^n} > 9 \rightarrow x^n < \frac{1}{9}$
 $x^n < \frac{1}{9} \rightarrow x < \sqrt[n]{\frac{1}{9}}$
 $D_f = \mathbb{R} - (-2, \sqrt[n]{\frac{1}{9}})$

ب) $\sqrt{x+1} = \sqrt{x} - \sqrt{x-1} \rightarrow \sqrt{x} - \sqrt{x-1} > 0 \rightarrow \sqrt{x} > \sqrt{x-1} \rightarrow x > x-1 \rightarrow x > 1$
 $D_f = [1, \infty)$

$x^2 - x - 2 > 0 \rightarrow (x+2)(x-1) > 0 \rightarrow x < -2 \text{ or } x > 1$
 $x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow x > 1 \text{ or } x < -1$
 $\sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1$
 $D_f = (-\infty, -1) \cup (1, \infty)$

$f(x) = \sqrt{x^2 + a} - x^2$
 $D_f = [-2, b]$
 $f(-2) = 0 \rightarrow \sqrt{4 + a} - 4 = 0 \rightarrow \sqrt{4 + a} = 4 \rightarrow 4 + a = 16 \rightarrow a = 12$
 $a + b = ?$
 $a = 12 \rightarrow f(x) = \sqrt{x^2 + 12} - x^2$
 $\sqrt{x^2 + 12} - x^2 = 0 \rightarrow \sqrt{x^2 + 12} = x^2 \rightarrow x^2 + 12 = x^4 \rightarrow x^4 - x^2 - 12 = 0$
 $x^2 = 4 \rightarrow x = \pm 2$
 $a + b = 12 + 12 = 24$

$g(x) = \sqrt{f(x)} - x$
 $f(x) - x > 0 \rightarrow f(x) > x$
 $\sqrt{x^2 + 12} - x^2 > x \rightarrow \sqrt{x^2 + 12} > x^2 + x$
 $x^2 + x < \sqrt{x^2 + 12}$
 $D_f = [-2, \infty)$

$$f(x) = 2x^2 - 1 \rightarrow f(2-1)$$

$$(2+\sqrt{2})(\sqrt{2}-2) = 2-2 = 0$$

$$f\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) \rightarrow f\left(\frac{\sqrt{2}}{2} + 1\right) \rightarrow f\left(\frac{\sqrt{2}+2}{2}\right) \rightarrow \sqrt{2} + 2\left(\frac{\sqrt{2}+2}{2} - 1\right) \rightarrow \sqrt{2} + \sqrt{2}(\sqrt{2}-1) = \frac{\sqrt{2}-1}{2} \left(-\frac{1}{2}\right)$$

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$$f(x) = \frac{x+1}{x-1} \rightarrow f(a) = \frac{a+1}{a-1} \xrightarrow{f(f(a))=a} f\left(\frac{a+1}{a-1}\right) = a \quad \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \rightarrow \frac{a+1+a-1}{a-1-a+1} = a \quad \frac{\frac{a}{a-1}}{\frac{1}{a-1}} = a$$

$a \in \mathbb{R}$

اینکه تکرار به قدر است.

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$$f(g(a)) = -1 \xrightarrow{g(a)=x} f(x) = -1 \rightarrow x^2 - 1 = 0 \rightarrow (x-1)(x+1) = 0 \rightarrow x = 1 \text{ or } x = -1 \rightarrow g(a) = 1 \text{ or } g(a) = -1 \rightarrow a^2 - 1 = 1 \text{ or } a^2 - 1 = -1 \rightarrow a^2 = 2 \text{ or } a^2 = 0 \rightarrow a = \pm \sqrt{2} \text{ or } a = 0$$

$$f(x) = x^2 - 1$$

$$g(x) = \sqrt{x^2 - 1}$$

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$$f(x+y) = x^2 - 2xy + y^2$$

$$f(x) = f(-x)$$

$$f(x) = x^2 - 2x + 1 \quad x \times \sqrt{x} = 1 \times 1$$

$$f(-x) = x^2 + 2x + 1$$

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$$f(\cos^2 x + \sin^2 x) = \tan^2 x$$

$$f(1) = ?$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x + \sin^2 x - \frac{1}{2} = 0$$

$$\cos^2 x = \frac{1}{2}$$

$$\frac{-1 \pm \sqrt{1 - 4 \cdot (-\frac{1}{2})}}{2 \cdot (-\frac{1}{2})} \rightarrow \frac{-1 \pm 1}{-1} \rightarrow \frac{0}{-1} = 0$$

$$\frac{1}{2} \sqrt{1 - 4 \cdot (-\frac{1}{2})}$$

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$$f(x) = x^2$$

$$x = 0, 2, 0 \quad \tan x = \sqrt{2}, -\sqrt{2}$$

$$\cos x = \frac{1}{2} \quad \tan x = \sqrt{2}$$

$$-1 \leq \cos x \leq 1$$

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