

۲. آزمون های نوبتی

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} = \sqrt{\frac{x^2 - 2x + 1}{x(x-1)}} = \sqrt{\frac{-2x+1}{x(x-1)}} \rightarrow x = \frac{1}{2}$

$\frac{0}{+} + \frac{1}{-} + \frac{1}{+} = - \Rightarrow x = (-\infty, 0) \cup [\frac{1}{2}, 1)$

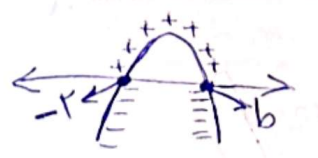
ب) $f(x) = \frac{1}{x+1} - \frac{x}{x-1} \Rightarrow \begin{cases} x \neq 1 \\ x \neq 0 \\ x \neq -2 \end{cases} \begin{cases} \frac{x}{x-1} \neq -1 \rightarrow 2x+1 \neq 1-x \\ \frac{x}{x-1} \neq -1 \rightarrow 2x+1 \neq 1-x \\ \frac{x}{x-1} \neq -1 \rightarrow 2x+1 \neq 1-x \end{cases} \Rightarrow D = \mathbb{R} - \{0, 1, -2, -\frac{5}{2}\}$

الف) $\sqrt{(\frac{1}{x})^2 - 9} \times (x^2 - 25)$
 $\downarrow \quad \downarrow$
 $x = -2 \quad x = 5 \rightarrow x = \frac{5}{2}$

$\frac{-2}{+} + \frac{0}{-} + \frac{0}{+} = - \Rightarrow D = \mathbb{R} - (-2, \frac{5}{2})$

ب) $\sqrt{x-1} + \sqrt{y+1} = 2 \rightarrow \sqrt{y+1} = 2 - \sqrt{x-1} \Rightarrow \sqrt{x-1} \leq 2$
 $0 \leq x-1 \leq 4 \Rightarrow 1 \leq x \leq 5$

$f(x) = \frac{\ln(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow x^2 - x - 2 > 0 \Rightarrow \frac{-1 \pm 2}{2} \Rightarrow \begin{cases} x < -1 \\ x > 2 \end{cases}$
 $\sqrt{x^2 - 1} + 1 \neq 0 \rightarrow x^2 - 1 \neq -1 \checkmark$
 $x^2 > 1 \Rightarrow \begin{cases} x > 1 \\ x < -1 \end{cases}$
 $D = (-\infty, -1) \cup (2, \infty)$

$f(x) = \sqrt{x^2 + ax + 4x^2} \xrightarrow{x=-2, y=0} -2 - 2a + 3 = 0 \rightarrow a = -\frac{1}{2}$

 $\rightarrow f(x) = \sqrt{x^2 + \frac{1}{2}x - 6}$
 $\alpha = -2 \rightarrow \beta = 3 \rightarrow b = \frac{3}{2}$
 $a+b = 1$

$g(x) = \sqrt{f(x) - x}$, $f(x) = \begin{cases} x-2 & x \geq 1 \\ 2x+6 & x < 1 \end{cases}$
 $\left. \begin{aligned} &\sqrt{2x-2-x} = \sqrt{x-2} \rightarrow x \geq 2 \\ &\sqrt{2x+3-x} = \sqrt{x+3} \rightarrow x \geq -3 \end{aligned} \right\} \Rightarrow D = [-3, \infty)$

$-3 \leq x < 1$

$$f(x) = x^2 - \varepsilon x = x(x - \varepsilon)$$

$$f\left(\sin \frac{\pi}{c} + \cos r\pi\right) = f\left(\frac{\sqrt{c}}{r} + 1\right)$$

$$\Rightarrow f\left(\frac{\sqrt{c} + r}{r}\right) = (\sqrt{c} + r)\left(\frac{\sqrt{c} + r}{r} - \varepsilon\right)$$

$$= (\sqrt{c} + r)\left(\frac{\sqrt{c}}{r} - 1\right) = \frac{1}{r}(\sqrt{c} - r)(\sqrt{c} + r) = \frac{1}{r}(c - \varepsilon) = \frac{1}{r}$$

$$f(x) = \frac{x+1}{x-1} \Rightarrow \frac{f(a)+1}{f(a)-1} = a \rightarrow af(a) - a = f(a) + 1$$

$$f(f(a)) = a \rightarrow f(a) = \frac{a+1}{a-1} \Rightarrow a \neq 1$$

$$D = \mathbb{R} - \{1\}$$

$$\textcircled{1} f(x) = x^2 - \varepsilon x \xrightarrow{\textcircled{1}, \textcircled{2}} g(x) - \varepsilon g(x) = -\varepsilon \Rightarrow g(x) = r$$

$$\textcircled{2} g(x) = \sqrt{x^2 - 1} \quad g(x) = r \rightarrow \sqrt{x^2 - 1} = r \rightarrow x = \pm \sqrt{r^2 + 1}$$

$$\textcircled{3} f(g(x)) = -\varepsilon$$

$$f(x+y) = x^2 - \Delta x + \gamma$$

$$f(\varepsilon) \times f(-\varepsilon) \Rightarrow \begin{cases} f(\varepsilon) : x=1 \Rightarrow f(\varepsilon) = 1 - \Delta + \gamma = r \\ f(-\varepsilon) : x=-\gamma \Rightarrow f(-\varepsilon) = \gamma^2 + \gamma + \gamma = \gamma^2 \end{cases} \Rightarrow \frac{f(\varepsilon)f(-\varepsilon)}{f(\varepsilon)f(-\varepsilon)} = \frac{1 \cdot \gamma^2}{\gamma^2} = 1$$

$$f(\varepsilon \cos^r u + \delta \cos u) = \tan^r u$$

$$\varepsilon \cos^r u + \delta \cos u - \frac{\gamma}{r} = 0 \rightarrow \cos u = \frac{-\delta \pm \sqrt{\delta^2 + 4\varepsilon\gamma}}{2\varepsilon}$$

$$\rightarrow \cos u = \frac{-\delta + \gamma}{\Lambda} = \frac{\varepsilon}{\Lambda} \Rightarrow \cos u = \frac{1}{r} \rightarrow \tan^r u = \frac{1}{\cos^r u} = r$$

$$\tan^r u = \frac{1}{\frac{1}{r}} = r$$