

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} = \sqrt{\frac{x^2 - 2x + 1}{x(x-1)}} = \sqrt{\frac{-2x+1}{x(x-1)}} \rightarrow x = \frac{1}{2}$
 $\frac{0}{+} \frac{1}{+} \frac{1}{+} \frac{1}{+} \Rightarrow x = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{1}{x+1} - \frac{x}{x-1} \Rightarrow \begin{cases} x \neq 1 \\ x \neq 0 \\ x \neq -2 \end{cases} \Rightarrow \frac{x}{x-1} \neq \frac{-1}{x+2} \rightarrow 2x+2 \neq 1-x \rightarrow 3x \neq -3 \rightarrow x \neq -1$
 $\frac{x}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow D = \mathbb{R} - \{0, 1, -2, -1\}$

الف) $\sqrt{(\frac{1}{x})^2 - 9} \times (x-5^x)$
 $\downarrow \quad \downarrow$
 $x = -2 \quad x = 5 \Rightarrow x = \frac{5}{2}$
 $\Rightarrow D = \mathbb{R} - (-2, \frac{5}{2})$

ب) $\sqrt{x-1} + \sqrt{y+1} = 2 \rightarrow \sqrt{y+1} = 2 - \sqrt{x-1} \Rightarrow \sqrt{x-1} \leq 2$
 $\Rightarrow 0 \leq x-1 \leq 4 \Rightarrow 1 \leq x \leq 5$

$f(x) = \frac{\log_e(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \Rightarrow x^2 - x - 2 > 0 \Rightarrow \frac{-1 \pm 2}{2} \Rightarrow \begin{cases} x < -1 \\ x > 2 \end{cases}$ (I)
 $\sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow x^2 - 1 \neq -1 \checkmark$
 $\Rightarrow x^2 > 1 \Rightarrow \begin{cases} x > 1 \\ x < -1 \end{cases}$ (II)
 $\Rightarrow D = (-\infty, -1) \cup (2, \infty)$

$f(x) = \sqrt{2 + ax + x^2} \xrightarrow{x=-2, y=0} -2 - 2a + 2 = 0 \Rightarrow a = -\frac{1}{2}$
 $\Rightarrow f(x) = \sqrt{-(x^2 + \frac{1}{2}x - 2)}$
 $\Rightarrow a+b = (-1) + 2 = 1$
 $\alpha \beta = -2 \xrightarrow{\alpha = -2} \beta = b = \frac{2}{-2} = -1$

$g(x) = \sqrt{f(x) - x}$, $f(x) = \begin{cases} x-2 & x \geq 1 \\ 2x+2 & x < 1 \end{cases}$

$g(x) \begin{cases} \sqrt{2x-2-x} = \sqrt{x-2} \rightarrow x \geq 2 \\ \sqrt{2x+2-x} = \sqrt{x+2} \rightarrow x \geq -2 \end{cases} \Rightarrow D = [-2, \infty)$

$-2 \leq x < 1$

$$f(x) = x^2 - \varepsilon x = x(x - \varepsilon)$$

$$f\left(\sin \frac{\pi}{c} + \cos r\pi\right) = f\left(\frac{\sqrt{c}}{r} + 1\right) \Rightarrow f\left(\frac{\sqrt{c}+r}{r}\right) = (\sqrt{c}+r)\left(\frac{\sqrt{c}+r}{r} - \varepsilon\right)$$

$$= (\sqrt{c}+r)\left(\frac{\sqrt{c}}{r} - 1\right) = \frac{1}{r}(\sqrt{c}+r)(\sqrt{c}-r) = \frac{1}{r}(c-\varepsilon) = \frac{1}{r}$$

$$\left. \begin{aligned} f(x) &= \frac{x+1}{x-1} \\ f(f(x)) &= a \end{aligned} \right\} \Rightarrow \frac{f(x)+1}{f(x)-1} = a \rightarrow af(x) - a = f(x) + 1$$

$$\rightarrow f(x) = \frac{a+1}{a-1} \Rightarrow a \neq 1$$

$$D = \mathbb{R} - \{1\}$$

$$\textcircled{1} f(x) = x^2 - \varepsilon x \xrightarrow{\textcircled{1}, \textcircled{2}} g(x) - \varepsilon g(x) = -\varepsilon \Rightarrow g(x) = r$$

$$\textcircled{2} g(x) = \sqrt{x^2 - 1} \quad g(x) = r \rightarrow \sqrt{x^2 - 1} = r \rightarrow x = \pm \sqrt{r^2 + 1}$$

$$\textcircled{3} f(g(x)) = -\varepsilon$$

$$f(x+y) = x^2 - \Delta x + \gamma$$

$$f(\varepsilon) \times f(-c) \Rightarrow \left\{ \begin{aligned} f(\varepsilon) : x=1 &\Rightarrow f(\varepsilon) = 1 - \Delta + \gamma = r \\ f(-c) : x=-\gamma &\Rightarrow f(-c) = \gamma^2 + \gamma + \gamma = \gamma^2 \end{aligned} \right\} \Rightarrow \frac{f(\varepsilon)f(-c)}{9} = \frac{1 \cdot \varepsilon}{9}$$

$$f(\varepsilon \cos^2 u + \delta \cos u) = \tan^2 u$$

$$\varepsilon \cos^2 u + \delta \cos u - \frac{\gamma}{r} = 0 \rightarrow \cos u = \frac{-\delta \pm \sqrt{\delta^2 + 4\varepsilon(\frac{\gamma}{r} - \frac{\delta^2}{4})}}{2\varepsilon}$$

$$\rightarrow \cos u = \frac{-\delta + \gamma}{\Delta} = \frac{\varepsilon}{\Delta} \Rightarrow \cos u = \frac{1}{r} \rightarrow \tan^2 u = \frac{1}{\cos^2 u} - 1$$

$$\rightarrow \frac{-\delta - \gamma}{\Delta} = \frac{-1 \cdot \varepsilon}{\Delta} \xrightarrow{-1 < \cos u < 1} \frac{1}{\varepsilon}$$

$$\tan^2 u = \frac{1}{\frac{1}{\varepsilon}} - 1 = \varepsilon$$