

۲۰ آذرین

الف) $\frac{a-1}{a} - \frac{a}{a-1} \geq 0 \Rightarrow \frac{(a-1)-a^2}{a(a-1)} \geq 0 \Rightarrow \frac{-a^2+a-1}{a(a-1)} \geq 0 \Rightarrow \frac{a^2-a-1}{a(a-1)} \leq 0$ ب) $a \neq 0, a \neq 1, a \neq -1, a \neq -2, \frac{3}{a-1} + \frac{1}{a+2} \neq 0 \Rightarrow \frac{3(a+2) + (a-1)}{(a-1)(a+2)} \neq 0$ $\Rightarrow 4a+5 \neq 0 \Rightarrow a \neq -\frac{5}{4}$ $\Rightarrow P_f = \mathbb{R} - \{0, 1, -2, -\frac{5}{4}\}$	۱
الف) $(3^{-n} - 9)(32 - 4^n) \geq 0 \Rightarrow \begin{cases} n = -2 \\ n = 5/2 \end{cases} \Rightarrow \frac{-2}{-4-4} \Rightarrow P_f = (-\infty, 2] \cup [5/2, +\infty)$ ب) $a-1 \geq 0 \Rightarrow a \geq 1, \sqrt{a+1} = 3 - \sqrt{a-1} \Rightarrow 3 - \sqrt{a-1} \geq 0 \Rightarrow \sqrt{a-1} \leq 3 \Rightarrow a-1 \leq 9 \Rightarrow a \leq 10$ $\Rightarrow 1 \leq a \leq 10 \Rightarrow P_f = [1, 10]$	۲
۱ $\Rightarrow \text{deg}(x^2 - x - 2) \Rightarrow x^2 - x - 2 \geq 0 \Rightarrow (x+1)(x-2) \geq 0 \Rightarrow x \in (-\infty, -1] \cup [2, +\infty)$ ۲ $\Rightarrow x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \geq 1, x \leq -1 \Rightarrow (-\infty, -1] \cup [1, +\infty)$ ۳ $\Rightarrow \sqrt{x^2 - 1} + 1 \neq 0$ $P_f = ① \cap ② = (-\infty, -1] \cup [2, +\infty)$	۳
..... $g(x) \rightarrow f(x) \geq x$ ① $x \geq 1 \Rightarrow x - \frac{1}{x} \geq x \Rightarrow 2x \geq 1 \Rightarrow x \geq \frac{1}{2} \Rightarrow [1, +\infty)$ ② $x < 1 \Rightarrow 2x + 3 \geq x \Rightarrow x \geq -3 \Rightarrow [-3, +\infty)$ $\Rightarrow P_f = ① \cup ② = [-3, +\infty)$	۵
$[-2, b] \rightarrow \sqrt{3-2a-4} = 0 \Rightarrow -1-2a=0 \Rightarrow a = -\frac{1}{2}$ $\hookrightarrow \sqrt{-a^2 + 4a + 3} \Rightarrow b + (-2) = 3 = \frac{b}{a} = -\frac{1}{2} \Rightarrow b = \frac{3}{2}$ $a + b = ①$	۴

$$f(m) = 2m^2 - 4m$$

$$f(\sqrt{2}\pi_3 + \cos 2\pi) = 2\left(\frac{\sqrt{2}}{2} + 1\right)^2 - 4\left(\frac{\sqrt{2}}{2} + 1\right)$$

$$\Rightarrow 2\left(\frac{\sqrt{2}}{2} + 1\right)^2 - 4\left(\frac{\sqrt{2}}{2} + 1\right) = \frac{7}{2} + 2\sqrt{2} - 2\sqrt{2} - 4 = -\frac{1}{2}$$

$$f(m) = \frac{m+1}{m-1} \rightarrow f(f(a))$$

$$= \frac{f(a)+1}{f(a)-1} = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1}$$

$$= \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a$$

$$Q \text{ } f \rightarrow a/1 \checkmark$$

$$a) f'(a) \neq 1 \rightarrow \text{not a fixed point}$$

$$a+1 = a-1 \Rightarrow 0 = -2$$

$$a \in \mathbb{R} - \{1\}$$

$$a \neq 1$$

$$\begin{cases} f(m) = m^2 - 4m \\ g(m) = \sqrt{m^2 - 1} \end{cases} \quad f(g(w)) = -4$$

$$f(z) = z^2 - 4z = -4 \Rightarrow z^2 - 4z + 4 = 0 \Rightarrow (z-2)^2 = 0 \Rightarrow z = 2$$

$$g(a) = 2 \Rightarrow \sqrt{a^2 - 1} = 2 \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow a = \pm\sqrt{5}$$

$$f(m+3) = m^2 - 5m + 6$$

$$f(4) \cdot f(-3) =$$

$$m=1$$

$$f(4) = 1 - 5 + 6 = 2$$

$$m=-6$$

$$f(-3) = 36 + 30 + 6 = 72$$

$$f(4) \cdot f(-3) = 2 \times 72 = 144$$

$$\cos m \left(4 \cos m + 5 \right) = \frac{7}{2} \Rightarrow \cos m = \frac{1}{2} \Rightarrow m = \frac{\pi}{3}$$

$$\Rightarrow f(3.5) = 2 \cos^2(\pi/3) = (\sqrt{2})^2 = 2$$