

الف) $\frac{a-1}{a} - \frac{a}{a-1} \geq 0 \Rightarrow \frac{(a-1)-a^2}{a(a-1)} \geq 0 \Rightarrow \frac{-a^2+a-1}{a(a-1)} \geq 0 \Rightarrow \frac{a^2-a-1}{a(a-1)} \leq 0$
 $\frac{a^2-a-1}{a(a-1)} \leq 0 \Rightarrow \frac{a^2-a-1}{a(a-1)} \leq 0 \Rightarrow \frac{a^2-a-1}{a(a-1)} \leq 0 \Rightarrow \frac{a^2-a-1}{a(a-1)} \leq 0$
 $\frac{a^2-a-1}{a(a-1)} \leq 0 \Rightarrow \frac{a^2-a-1}{a(a-1)} \leq 0 \Rightarrow \frac{a^2-a-1}{a(a-1)} \leq 0$

ب) $a \neq 0, a \neq 1, a \neq -1, a \neq -2, \frac{3}{a-1} + \frac{1}{a+2} \neq 0 \Rightarrow \frac{3(a+2) + (a-1)}{(a-1)(a+2)} \neq 0$

$\Rightarrow 3a+6+a-1 \neq 0 \Rightarrow 4a+5 \neq 0 \Rightarrow a \neq -\frac{5}{4}$
 $\Rightarrow P_f = \mathbb{R} - \{0, 1, -1, -2, -\frac{5}{4}\}$

الف) $(3^m - 9)(32 - 4^m) \geq 0 \Rightarrow \begin{cases} m \geq 2 \\ m \leq \frac{5}{2} \end{cases} \Rightarrow P_f = (-\infty, 2] \cup [\frac{5}{2}, +\infty)$

ب) $a-1 \geq 0 \Rightarrow a \geq 1, \sqrt{a+1} = 3 - \sqrt{a-1} \Rightarrow \sqrt{a+1} + \sqrt{a-1} = 3 \Rightarrow \sqrt{a+1} \geq 3 - \sqrt{a-1}$
 $\Rightarrow \sqrt{a+1} \geq 3 - \sqrt{a-1} \Rightarrow \sqrt{a+1} \geq 3 - \sqrt{a-1} \Rightarrow \sqrt{a+1} \geq 3 - \sqrt{a-1}$
 $\Rightarrow \sqrt{a+1} \geq 3 - \sqrt{a-1} \Rightarrow \sqrt{a+1} \geq 3 - \sqrt{a-1} \Rightarrow \sqrt{a+1} \geq 3 - \sqrt{a-1}$

1- $\text{deg}(a^2 - a - 2) \rightarrow a^2 - a - 2 \geq 0 \Rightarrow (a+1)(a-2) \geq 0 \Rightarrow \frac{-1}{-1} \leq a \leq \frac{2}{2} \Rightarrow (-\infty, -1] \cup [2, +\infty)$

2- $a^2 - 1 \geq 0 \Rightarrow a^2 \geq 1 \Rightarrow a \geq 1, a \leq -1 \Rightarrow (-\infty, -1] \cup [1, +\infty)$

3- $\sqrt{a^2 - 1} + 1 \neq 0$
 $P_f = ① \cap ② = (-\infty, -1) \cup (2, +\infty)$

~~g(x) \rightarrow f(x) \geq x~~
 $g(x) \rightarrow f(x) \geq x$
 $① \Rightarrow x \geq 1 \Rightarrow 3x - 2 \geq x \Rightarrow 2x \geq 2 \Rightarrow x \geq 1 \Rightarrow [1, +\infty)$
 $② \Rightarrow x < 1 \Rightarrow 2x + 3 \geq x \Rightarrow x \geq -3 \Rightarrow [-3, +\infty)$
 $\Rightarrow P_f = ① \cup ② = [-3, +\infty)$

$[-2, b] \rightarrow \sqrt{3-2a-4} = 0 \Rightarrow -1-2a=0 \Rightarrow a = -\frac{1}{2}$

$\rightarrow \sqrt{-a^2 + 4a + 3} \Rightarrow b + (-2) = 3 = \frac{b}{a} = -\frac{1}{2} \Rightarrow b = \frac{3}{2}$

$a + b = ①$

$$f(m) = 2m^2 - 4m$$

$$f(\sqrt{2}\pi_3 + \cos 2\pi) = 2\left(\frac{\sqrt{2}}{2} + 1\right)^2 - 4\left(\frac{\sqrt{2}}{2} + 1\right)$$

$$\Rightarrow 2\left(\frac{\sqrt{2}}{2} + 1\right)^2 - 4\left(\frac{\sqrt{2}}{2} + 1\right) = \frac{7}{2} + 2\sqrt{2} - 2\sqrt{2} - 4 = \boxed{-\frac{1}{2}}$$

$$f(m) = \frac{m+1}{m-1} \rightarrow f(f(a))$$

$$= \frac{f(a)+1}{f(a)-1} = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1}$$

$$= \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a$$

$$Q \text{ } f \rightarrow a/1 \checkmark$$

$$(a) f'(a) \neq 1 \rightarrow \text{not a fixed point}$$

$$a+1 = a-1 \Rightarrow 0 = -2$$

$$a \in \mathbb{R} - \{1\}$$

$$\begin{cases} f(m) = m^2 - 4m \\ g(m) = \sqrt{m^2 - 1} \end{cases} \quad f(g(w)) = -4$$

$$f(z) = z^2 - 4z = -4 \Rightarrow z^2 - 4z + 4 = 0 \Rightarrow (z-2)^2 = 0 \Rightarrow z = 2$$

$$g(a) = 2 \Rightarrow \sqrt{a^2 - 1} = 2 \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow a = \pm\sqrt{5}$$

$$f(m+3) = m^2 - 5m + 6$$

$$f(4) \cdot f(-3) =$$

$$m=1 \quad f(4) = 1 - 5 + 6 = 2$$

$$m=-6 \quad f(-3) = 36 + 30 + 6 = 72$$

$$f(4) \cdot f(-3) = 2 \times 72 = 144$$

$$\cos m \left(4 \cos m + 5 \right) = \frac{7}{2} \Rightarrow \cos m = \frac{1}{2} \Rightarrow m = \frac{\pi}{3}$$

$$\Rightarrow f(3.5) = 2 \cos^2(\pi/3) = (\sqrt{3})^2 = 3$$