

۱۹،۵ آفرین

نام و نام خانوادگی رازدار پاسخنامه تشریحی تکلیف شماره کلاس از ۵۰

الف $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} = \sqrt{\frac{x^2-2x+1-x^2}{x^2(x-1)}} = \sqrt{\frac{-1(x-1)}{x^2(x-1)}} = \sqrt{\frac{-1}{x^2}}$ $\Rightarrow$ $\frac{1}{x^2} \geq 0 \Rightarrow x \neq 0$ $\Rightarrow$ $D_f = (-\infty, 0) \cup (0, \infty)$

$\frac{3}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow \frac{3(x+2) + x-1}{(x-1)(x+2)} \neq 0 \Rightarrow 4x+5 \neq 0 \Rightarrow x \neq -\frac{5}{4}$

ب $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x+1} + \frac{1}{x}} = \frac{\frac{x - (x+1)}{x(x+1)}}{\frac{x + (x+1)}{x(x+1)}} = \frac{-1}{2x+1}$ $D_f = \mathbb{R} - \{-\frac{1}{2}\}$

ج $f(x) = \sqrt{(\frac{1}{x}-9)(\frac{1}{x^2}-\frac{1}{x})}$ $\Rightarrow \frac{1}{x} - 9 \geq 0 \Rightarrow \frac{1}{x} \geq 9 \Rightarrow x \leq \frac{1}{9}$ $\Rightarrow \frac{1}{x^2} - \frac{1}{x} \geq 0 \Rightarrow \frac{1-x}{x^2} \geq 0 \Rightarrow 1-x \geq 0 \Rightarrow x \leq 1$ $\Rightarrow D_f = (-\infty, \frac{1}{9}] \cup [\frac{1}{10}, \infty)$

$D_f = (1, 12]$

ب $\sqrt{x-1} + \sqrt{y+1} = 2$ $\Rightarrow \sqrt{x-1} = 2 - \sqrt{y+1}$ $\Rightarrow x-1 = 4 - 4\sqrt{y+1} + y+1 \Rightarrow x = y + 4 - 4\sqrt{y+1}$

$f(x) = \frac{\log(x^2-x-2)}{\sqrt{x^2-1}+1}$

$x^2-x-2 \geq 0 \Rightarrow x \leq -1$ or $x \geq 2$ $\Rightarrow D_f = (-\infty, -1] \cup [2, \infty)$

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سپیدار یکال صوره $\Rightarrow$ $x > 2 \wedge x \geq 1 \Rightarrow x > 2$

$f(x) = \sqrt{-x^2+ax+1}$ و $D_f = [-2, b]$

$-1 \rightarrow -(-2)^2 + a(-2) + 1 = 0 \Rightarrow -2a = 1 \Rightarrow a = -\frac{1}{2} \Rightarrow f(x) = \sqrt{-x^2 + \frac{1}{2}x + 1}$

$p = \frac{c}{a} \Rightarrow -2 = \frac{-2xb}{\frac{1}{2}} \Rightarrow b = \frac{2}{3}$ $a+b = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

چون -2 ریشه انتگرال دانه است پس می توان اینطور استنتاج کرد که -2 ریشه دیگر را یکال بودن است

$f(x) = \sqrt{f(x)}x$ $f(x) \begin{cases} 3x-2 & x \geq 1 \\ 4x+3 & x < 1 \end{cases}$

$f(x) - x \geq 0$ $\Rightarrow \begin{cases} 3x-2-x \geq 0 \Rightarrow 2x-2 \geq 0 \Rightarrow x \geq 1 \\ 4x+3-x \geq 0 \Rightarrow 3x+3 \geq 0 \Rightarrow x \geq -1 \end{cases} \Rightarrow D_f = [-1, \infty)$

$$f(x) = x^2 - \varepsilon x$$

$$f\left(\frac{\sqrt{x} + 1}{2}\right) = f\left(\frac{\sqrt{x} + 1}{2}\right) = \left(\frac{\sqrt{x} + 1}{2}\right)^2 - \varepsilon \left(\frac{\sqrt{x} + 1}{2}\right) = \frac{x + 2\sqrt{x} + 1}{4} - \varepsilon \frac{\sqrt{x} + 1}{2} = \frac{x + 2\sqrt{x} + 1 - \varepsilon\sqrt{x} - \varepsilon}{4} = \frac{x + 2\sqrt{x} + 1 - \varepsilon\sqrt{x} - \varepsilon}{4}$$

$$f(x) = x^2 - \varepsilon x \quad g(x) = \sqrt{x-1} \quad f(g(0)) = -\varepsilon$$

$$f(g(x)) = (\sqrt{x-1})^2 - \varepsilon \sqrt{x-1} = x - \varepsilon \sqrt{x-1} = -\varepsilon \quad \sqrt{x-1} = t \Rightarrow x = t^2 + 1 \Rightarrow x-1 = t^2 \Rightarrow x = t^2 + 1$$

$$f(x) = \frac{x+1}{x-1} \quad f(f(a)) = a$$

$$f(a) = \frac{a+1}{a-1} \Rightarrow a = f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a \Rightarrow a = a$$

$$f(x) = x^2 - 0x + 9$$

$$x+1 \neq t \quad \left. \begin{array}{l} x=1 \Rightarrow f(1) = 1^2 - 0(1) + 9 = 10 \\ x=-1 \Rightarrow f(-1) = (-1)^2 - 0(-1) + 9 = 10 \end{array} \right\} \Rightarrow f(1) \times f(-1) = 10 \times 10 = 100$$

$$f(\cos x) = \cos^2 x = \tan^2 x \quad \cos x = \frac{1}{2} \Rightarrow \cos^2 x = \frac{1}{4}$$

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x + \frac{1}{4} = 1 \Rightarrow \sin^2 x = \frac{3}{4} \Rightarrow \sin x = \frac{\sqrt{3}}{2}$$

تمام ریشه‌های مخرج‌ها نباید در دامنه باشند! $x+1 \neq 0 \rightarrow x \neq -1$ $x \neq 0$ $x-1 \neq 0 \rightarrow x \neq 1$

$$x+2 \neq 0 \rightarrow x \neq -2 \quad \frac{x^3}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{x^3 + x + 1}{(x-1)(x+2)} \neq 0 \rightarrow x^3 + x + 1 \neq 0 \rightarrow x \neq -\frac{1}{x^2}$$

$$D_f = \mathbb{R} - \left\{ -1, 0, 1, -2, -\frac{1}{x^2} \right\}$$

$$3- \log_{\epsilon} x^2 - x - 2 \rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \quad \frac{-1 \pm 2}{+ \phi - \phi +} \rightarrow x > 2 \text{ یا } x < -1$$

$$\sqrt{x^2-1} + 1 \rightarrow \text{عوارض مثبت است} \rightarrow \sqrt{x^2-1} \rightarrow x^2-1 \geq 0 \rightarrow x \geq 1 \text{ یا } x \leq -1$$

$$I, II \rightarrow x < -1, x > 2 \rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$$