

جواب نهایی قسمت الف سؤال ۲: $D_f = (-\infty, -2] \cup [\frac{5}{4}, +\infty)$ است (از صورت عدم وضوح)

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الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{x^2 - (x+1) - x^2}{x^2 - x}} \Rightarrow \frac{x}{f(x)} = \frac{1}{\frac{1}{x} - \frac{1}{x-1}}$

$\Rightarrow D_f = (-\infty, 0) \cup (\frac{1}{2}, 1)$

$x+1 \neq 0 \Rightarrow x \neq -1$ ① $x \neq 0$ ② $x-1 \neq 0 \Rightarrow x \neq 1$ ③

ب) $x+2 \neq 0 \Rightarrow x \neq -2$ ④ $\frac{x}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow \frac{x^2 + x - 2}{x^2 - x} \neq 0 \Rightarrow x \neq -\frac{5}{4}$ ⑤

$\Rightarrow D_f = \mathbb{R} - \{-2, -\frac{5}{4}, 1\}$

الف) $(\frac{1}{x})^x - 9 = 0 \Rightarrow x = -2$

$x^2 - x^x = 0 \Rightarrow x = \frac{1}{2} \Rightarrow \frac{x}{f(x)} = \frac{-2}{\frac{1}{2}}$

ب) $\sqrt{y+1} = \sqrt[3]{x-1} \Rightarrow \sqrt[3]{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq \sqrt[3]{x-1}$ ① ②

$\Rightarrow x-1 \leq 1 \Rightarrow x \leq 2$ ① $x-1 \geq 0 \Rightarrow x \geq 1$ ② $\Rightarrow D_f = [1, 2]$

$x^2 - x - 2 > 0 \Rightarrow \begin{cases} x > 2 \\ x < -1 \end{cases}$ ① ② $\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow \begin{cases} x \geq 1 \\ x \leq -1 \end{cases}$ ③ ④

$\sqrt{x^2 - 1} \neq -1$ همیشه برقرار است

$f(-2) = 0 \Rightarrow \sqrt[3]{-2} - \frac{1}{2} = 0 \Rightarrow a = -\frac{1}{2}$

$\Rightarrow f(x) = \sqrt{-x^2 - \frac{1}{2}x + \frac{3}{2}} = \sqrt{(x+2)(\frac{3}{2} - x)}$

$\Rightarrow D_f = [-2, \frac{3}{2}] \Rightarrow b = \frac{3}{2} \Rightarrow a+b = -\frac{1}{2} + \frac{3}{2} = 1$ ①

$f(x) - x \geq 0 \Rightarrow f(x) \geq x \Rightarrow \begin{cases} \text{if } x \geq 1 \Rightarrow \sqrt{x^2 - 2} \geq x \Rightarrow x \geq 1 \\ \text{if } x < 1 \Rightarrow \sqrt{x^2 + 3} \geq x \Rightarrow x \geq -3 \end{cases}$ ① ②

$\Rightarrow D_g = [-3, 1) \cup [1, +\infty) = [-3, +\infty)$

$$f(x) = x(x-1) \Rightarrow f\left(\sin \frac{\pi}{n} + \cos^2 \pi\right) = f\left(\frac{\sqrt{n}}{n} + 1\right) = \sqrt{n} + n \left(\frac{\sqrt{n}}{n} - 1\right) = \left(-\frac{1}{n}\right)$$

5

$$f(a) = \frac{a+1}{a-1} \Rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a$$

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$a \neq 1 \Rightarrow \frac{f(a)}{a} = a \Rightarrow$ به ازای تمام مقادیر $a \neq 1$ به جز
 $a = 1$ معادله $f(f(a)) = a$ ^{اصحیح} برقرار است $\Rightarrow a \in \mathbb{R} - \{1\}$

$$\begin{aligned} g(a) &= \sqrt{a^r - 1} & f(x) &= x^r - \varepsilon x \Rightarrow f(g(a)) = \\ a^r - 1 - \varepsilon \sqrt{a^r - 1} &= -\varepsilon \xrightarrow{\sqrt{a^r - 1} = t} \\ t^r - \varepsilon t + \varepsilon &= 0 \Rightarrow (t - 1)^r = 0 \Rightarrow t = 1 \Rightarrow \\ a^r - 1 &= \varepsilon \Rightarrow a^r = 1 + \varepsilon \Rightarrow a = \sqrt[r]{1 + \varepsilon} \end{aligned}$$

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$$f(x) = f(1+x^2) = 1 - 2 + 9 = 8$$

$$f(-w) = f(-g+w) = w^2 + w_0 + g - \sqrt{w^2 + w_0 + g}$$

$$\Rightarrow f(\varepsilon) \times f(-\varepsilon) = 2 \times 1/2 = 1$$

9

$$\cos x = x \Rightarrow f(x + \Delta x) \approx \frac{1}{x} - 1$$

$$\begin{aligned} \nabla \alpha^r + \partial \alpha &= \mu, \partial \Rightarrow \nabla \alpha^r + \partial \alpha - \mu, \partial = 0 \quad \text{در نظر} \\ (\alpha + r)(\alpha - r) &= 0 \quad \text{برای } -1 \leq \alpha \leq 1 \Rightarrow \alpha = \frac{r}{r-1} = \frac{1}{\frac{1}{r}-1} \end{aligned}$$

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