

جواب نظایر قسمت الف سؤال ۲ از کتاب (۱) است (۱) صورت عدم وجود

نام و نام خانوادگی کلاس پاسخنامه تشریحی تکلیف شماره کلاس B

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{x^2 - (x+1) - x^2}{x^2 - x}} = \sqrt{\frac{-x-1}{x^2 - x}} = \sqrt{\frac{x}{x^2 - x} - \frac{1}{x^2 - x}}$

$\Rightarrow D_f = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$

$x+1 \neq 0 \Rightarrow x \neq -1$ ① $x \neq 0$ ② $x-1 \neq 0 \Rightarrow x \neq 1$ ③

ب) $x+2 \neq 0 \Rightarrow x \neq -2$ ④ $\frac{x}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow \frac{x(x+2) + (x-1)}{(x-1)(x+2)} \neq 0 \Rightarrow x \neq -\frac{5}{2}$ ⑤

$\Rightarrow D_f = \mathbb{R} - \left\{-2, -\frac{5}{2}, -1, 0, 1\right\}$

الف) $\left(\frac{1}{x}\right)^x - 9 = 0 \Rightarrow x = -2$ $x^2 - x^x = 0 \Rightarrow x = \frac{1}{2} \Rightarrow \frac{x}{x^2 - x^x} = \frac{-2}{\frac{1}{4} - \frac{1}{2}} = \frac{-2}{-\frac{1}{4}} = 8$

ب) $\sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$ ① $x-1 \geq 0 \Rightarrow x \geq 1$ ② $\Rightarrow D_f = [1, 10]$

$x^2 - x - 2 > 0 \Rightarrow \begin{cases} x > 2 \\ x < -1 \end{cases}$ ① $\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow \begin{cases} x \geq 1 \\ x \leq -1 \end{cases}$ ②

$\sqrt{x^2 - 1} \neq -1$ هواره برقرار است

$f(-2) = 0 \Rightarrow x^2 - 2x - 6 = 0 \Rightarrow x = -\frac{1}{2}$

$\Rightarrow f(x) = \sqrt{-x^2 - \frac{1}{2}x + 3} = \sqrt{(x+2)\left(\frac{3}{2} - x\right)}$

$\Rightarrow D_f = \left[-2, \frac{3}{2}\right] \Rightarrow b = \frac{3}{2} \Rightarrow a + b = -\frac{1}{2} + \frac{3}{2} = 1$ ①

$f(x) - x \geq 0 \Rightarrow f(x) \geq x \Rightarrow \begin{cases} \text{if } x \geq 1 \Rightarrow x^2 - 2 \geq x \Rightarrow x \geq 1 \\ \text{if } x < 1 \Rightarrow x^2 + 3 \geq x \Rightarrow x \geq -3 \end{cases}$ ① ②

$\Rightarrow D_g = [-3, 1) \cup [1, +\infty) = [-3, +\infty)$

$$f(x) = x(x-1) \Rightarrow f\left(\sin \frac{\pi}{\sqrt{2}} + \cos \frac{\pi}{\sqrt{2}}\right) = f\left(\frac{\sqrt{2}}{\sqrt{2}} + 1\right) = \sqrt{2} + 1 \left(\frac{\sqrt{2}}{\sqrt{2}} - 1\right) = \left(-\frac{1}{\sqrt{2}}\right)$$

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$$f(a) = \frac{a+1}{a-1} \Rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = \frac{2a}{2} = a$$

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$a \neq 1 \Rightarrow \frac{2a}{2} = a \Rightarrow$ $\frac{a+1}{a-1} = 1$ $\Rightarrow$ $a+1 = a-1 \Rightarrow 2 = 0$ (مناقضه)
 $a = 1$ معادله $f(f(a)) = a$ برقرار است
 $a \in \mathbb{R} - \{1\}$

$$g(a) = \sqrt{a^2 - 1} \quad f(x) = x^2 - x \Rightarrow f(g(a)) = \sqrt{a^2 - 1} - \sqrt{a^2 - 1} = 0$$

$$a^2 - 1 = x \sqrt{a^2 - 1} \Rightarrow \sqrt{a^2 - 1} = t \Rightarrow t^2 - t + x = 0 \Rightarrow (t-1)^2 = 0 \Rightarrow t = 1 \Rightarrow \sqrt{a^2 - 1} = 1 \Rightarrow a^2 - 1 = 1 \Rightarrow a^2 = 2 \Rightarrow a = \pm \sqrt{2}$$

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$$f(x) = f(1+x) = 1 - x + 9 = 8 - x$$

$$f(-x) = f(-9+x) = -x + 9 + 9 = 18 - x$$

$$\Rightarrow f(x) \times f(-x) = (8-x)(18-x) = 144$$

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$$\cos x = \alpha \Rightarrow f\left(\frac{1}{\alpha^2} + \alpha\right) = \frac{1}{\alpha^2} - 1$$

$$\frac{1}{\alpha^2} + \alpha = 1, \alpha \Rightarrow \frac{1}{\alpha^2} + \alpha - 1 = 0 \Rightarrow \frac{1 + \alpha^3 - \alpha^2}{\alpha^2} = 0 \Rightarrow 1 + \alpha^3 - \alpha^2 = 0 \Rightarrow (1 + \alpha)(1 - \alpha) = 0 \Rightarrow \alpha = 1 \text{ or } \alpha = -1$$

$$\Rightarrow f(1, 1) = \frac{1}{1^2} - 1 = 0$$

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