

نام و نام خانوادگی پاسخنامه تشریحی تکلیف شماره ... کلاس ...

$$\text{الف) } \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x^2 - x} \geq 0 \Rightarrow \frac{-2x+1}{x(x-1)} \geq 0 \rightarrow \frac{0}{+} \frac{+}{-} \frac{+}{+} \frac{+}{-} \Rightarrow D_f = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$$

$$\begin{aligned} \text{ب) } x+1 \neq 0 &\Rightarrow x \neq -1 \\ x \neq 0 \\ \frac{x}{x-1} + \frac{1}{x+3} \neq 0 &\Rightarrow \frac{x^2 + 1}{x^2 + 2x - 3} \neq 0 \Rightarrow x \neq -3, 1, -2 \end{aligned} \Rightarrow D_f = \mathbb{R} - \{-3, -2, -1, 0, 1\}$$

$$\text{الف) } \left(\frac{1}{x}\right)^x - 9 = 0 \Rightarrow \left(\frac{1}{x}\right)^x = 9 \Rightarrow \frac{1}{x^x} = 9 \Rightarrow 1 = 9x^x \Rightarrow x^x = \frac{1}{9} \Rightarrow x^x = 9^{-1} \Rightarrow x = -2$$

$$x^x - 9 = 0 \Rightarrow x^x = 9 \Rightarrow x^x = 3^2 \Rightarrow x^x = 3^2 \Rightarrow x = \frac{2}{3}$$

$$\left(\left(\frac{1}{x}\right)^x - 9\right)(x^x - 9) \geq 0 \rightarrow \frac{-2}{+} \frac{\frac{2}{3}}{-} \Rightarrow D_f = \mathbb{R} - \left(2, \frac{2}{3}\right)$$

$$\begin{aligned} \text{ب) } x-1 > 0 &\Rightarrow x > 1 \\ x^x - 9 > 0 &\Rightarrow x^x > 9 \Rightarrow x^x > 3^2 \Rightarrow x > 2 \\ x-1 < 0 &\Rightarrow x < 1 \\ x^x - 9 < 0 &\Rightarrow x^x < 9 \Rightarrow x^x < 3^2 \Rightarrow x < 2 \end{aligned} \Rightarrow D_f = [1, 2]$$

$$\begin{aligned} x^x - x - 2 > 0 &\Rightarrow \frac{-1}{+} \frac{2}{+} \Rightarrow x = \mathbb{R} - [-1, 2] \\ x^x - 1 > 0 &\Rightarrow x^x > 1 \Rightarrow 1 < x < -1 \\ \sqrt{x^x - 1} + 1 \neq 0 &\Rightarrow \sqrt{x^x - 1} \neq -1 \rightarrow \text{همیشه برقرار} \end{aligned} \Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$$

$$\begin{aligned} -x^2 + ax + 3 &\xrightarrow{x=-2} -4 - 2a + 3 = 0 \Rightarrow -2a - 1 = 0 \Rightarrow -2a = 1 \Rightarrow a = -\frac{1}{2} \\ -x^2 - \frac{x}{2} + 3 &\geq 0 \Rightarrow \Delta = b^2 - 4ac = \frac{1}{4} + 12 = \frac{49}{4} \Rightarrow \sqrt{\Delta} = \sqrt{\frac{49}{4}} = \frac{7}{2} = 3.5 \\ \rightarrow x = \frac{0.5 \pm 3.5}{-2} &\Rightarrow x = -2 \text{ (بزرگتر)} \\ &\Rightarrow \frac{3}{2} = b \end{aligned} \Rightarrow a + b = -\frac{1}{2} + \frac{3}{2} = 1$$

$$f(x) - x \geq 0 \Rightarrow \begin{cases} x \geq 1 \Rightarrow 2x - 2 - x \geq 0 \Rightarrow x - 2 \geq 0 \Rightarrow x - 1 \geq 0 \Rightarrow x \geq 1 \text{ (I)} \\ x < 1 \Rightarrow 2x + 3 - x \geq 0 \Rightarrow x + 3 \geq 0 \Rightarrow x \geq -3 \text{ (II)} \end{cases}$$

$$\text{I} \cup \text{II} = D_g = [-3, +\infty)$$

$$f(x) = x^2 - 1 \Rightarrow x(x-1) \Rightarrow (\sqrt{x+1})\left(\frac{\sqrt{x-1}}{x}\right) = \frac{x-1}{x} = \boxed{\frac{-1}{x}}$$

$$\sin \frac{\pi}{x} + \cos 2\pi = \frac{\sqrt{x}}{x} + 1 = \frac{\sqrt{x+1}}{x}$$

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$$f(x) = a \Rightarrow \frac{x+1}{x-1} = a \Rightarrow x = \frac{-a-1}{a-1} = \frac{a+1}{a-1} = f(a) \Rightarrow \frac{a+1}{a-1} = \frac{a+1}{a-1}$$

$$f(a) = \frac{a+1}{a-1}$$

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$$\Rightarrow a = \mathbb{R} - \{1\}$$

$$x^2 - 1 = -1 \Rightarrow x = 1 = g(a)$$

$$\Rightarrow g(a) = 1 \Rightarrow \sqrt{a^2 - 1} = 1 \Rightarrow a^2 - 1 = 1 \Rightarrow a^2 = 2 \Rightarrow a = \boxed{\pm \sqrt{2}}$$

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$$x + 2 = 1 \Rightarrow x = -1 \rightarrow 1 - 2 + 4 = 1 = f(1)$$

$$x + 2 = -1 \Rightarrow x = -3 \rightarrow (-3)^2 - 2(-3) + 4 = 9 + 6 + 4 = 19 = f(-3)$$

$$\left. \begin{array}{l} x + 2 = 1 \Rightarrow x = -1 \rightarrow 1 - 2 + 4 = 1 = f(1) \\ x + 2 = -1 \Rightarrow x = -3 \rightarrow (-3)^2 - 2(-3) + 4 = 9 + 6 + 4 = 19 = f(-3) \end{array} \right\} \Rightarrow f(1) \cdot f(-3) = 1 \times 19 = \boxed{19}$$

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برای دیگر حالات قابل قبول در موردی نیستند

$$2 \cos^2 x + 2 \cos x = \left(2 \cos x + \frac{2}{x}\right)^2 - \frac{2}{x^2}$$

$$\Rightarrow \left(2 \cos x + \frac{2}{x}\right)^2 - \frac{2}{x^2} = \frac{9}{4} \Rightarrow \left(2 \cos x + \frac{2}{x}\right)^2 = \frac{9}{4} + \frac{2}{x^2} = \frac{11}{4} \Rightarrow 2 \cos x + \frac{2}{x} = \frac{\sqrt{11}}{2}$$

$$\Rightarrow 2 \cos x = \frac{\sqrt{11}}{2} - \frac{2}{x} \Rightarrow \cos x = \frac{\sqrt{11}}{4} - \frac{1}{x} \Rightarrow x = \frac{\pm \pi}{3} \xrightarrow{x = \frac{\pi}{3}} \tan^2 \frac{\pi}{3} = \frac{\sin^2 \frac{\pi}{3}}{\cos^2 \frac{\pi}{3}} = \frac{\frac{3}{4}}{\frac{1}{4}} = \boxed{3}$$

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در حالتی که x برابر $\frac{\pi}{3}$ است چون تقادیر به توان می ریزد، تفاوتی در جواب ایجاد نمی کند.