

$$\text{الف) } f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \quad \frac{x-1}{x} - \frac{x}{x-1} = \frac{(x-1)^2 - x}{x(x-1)} = \frac{-(2x-1)}{x(x-1)} \quad \begin{matrix} \nearrow \frac{1}{x} \\ \searrow 1 \end{matrix}$$

$$\begin{matrix} 0 & \frac{1}{2} & 1 \\ + & - & + \\ \frac{1}{2} & - & \frac{1}{2} \end{matrix} \quad Df = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$$

$$\text{ب) } \frac{x-2x-2}{(x+1)x} = \frac{-x-2}{x(x+1)} \Rightarrow x \rightarrow 0, -1 \quad \left| \frac{3x+4x-1}{(x-1)(x+2)} = \frac{f(x)+0}{(x-1)(x+2)} \Rightarrow x \rightarrow -\frac{2}{f}, -2, 1 \right.$$

$$Df = R - \left\{ 0, -1, -\frac{2}{f}, -2, 1 \right\}$$

$$\text{الف) } (3^{-x} - 9)(3^x - 4^x) \Rightarrow 3^{-x} - 9 = 0 \Rightarrow 3^{-x} = 9 \Rightarrow 3^{-x} = 3^2 \Rightarrow -x = 2 \Rightarrow x = -2$$

$$Df = (-\infty, -2] \cup \left[\frac{2}{f}, +\infty\right) \Rightarrow 3^x - 4^x = 0 \Rightarrow 4^x = 3^x \Rightarrow x = \log \frac{3^x}{4^x} = \frac{-2}{f} \quad \begin{matrix} -2 \\ + & - & + \\ \frac{2}{f} & - & \frac{2}{f} \end{matrix}$$

$$\text{ب) } 3 - \sqrt{x-1} = \sqrt{y+1} \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1 \rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$$

$$Df = [1, 10]$$

$$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \quad x^2 - x - 2 > 0 \Rightarrow (x+1)(x-2) > 0 \Rightarrow D = (-\infty, -1) \cup (2, +\infty)$$

$$\text{صفر نمی شود} \quad \text{① } x^2 - 1 \geq 0 \Rightarrow (x-1)(x+1) \geq 0 \Rightarrow (-\infty, -1] \cup [1, +\infty)$$

$$\text{①} \cap \text{②} \Rightarrow Df = (-\infty, -1) \cup (2, +\infty)$$

$$f(x) = \sqrt{r+ax-x^2} \Rightarrow \sqrt{r-ra} \cdot f = 0 \Rightarrow -1-ra = 0 \Rightarrow ra = -1 \Rightarrow a = -\frac{1}{r}$$

$$p = \frac{c}{A} \Rightarrow x_1 \cdot x_2 = \frac{r}{-1} \Rightarrow -rb = -r \Rightarrow b = \frac{r}{r}$$

$$Ax^2 + Bx + C = x^2 - \frac{1}{r}x + r \Rightarrow a+b = \frac{r}{r} - \frac{1}{r} = \textcircled{1}$$

$$\text{① } x \geq 1 \Rightarrow f(x) - x = 2x - 2 - x = x - 2 \quad \begin{matrix} 2x-2 \geq 0 \\ 2x \geq 2 \\ x \geq 1 \end{matrix}$$

$$\text{② } x < 1 \Rightarrow f(x) - x = 2x + 2 - x = x + 2 \quad \begin{matrix} x+2 \geq 0 \\ x \geq -2 \end{matrix}$$

$$\text{①} \cup \text{②} = [-2, +\infty)$$

$$f(x) = x^2 - 8x \Rightarrow f\left(\frac{\sqrt{r}}{r} + 1\right) = r\left(\frac{\sqrt{r}}{r} + 1\right)^2 - 8\left(\frac{\sqrt{r}}{r} + 1\right) - 4$$

$$\rightarrow r\left(\frac{\sqrt{r}}{r} + 1\right)^2 = r \times \left(\frac{r}{r} + 1 + \sqrt{r}\right) = r\left(\frac{r}{r} + \sqrt{r}\right) = r + r\sqrt{r}$$

$$\rightarrow -8\left(\frac{\sqrt{r}}{r} + 1\right) = -8\sqrt{r} - 8$$

$$r + r\sqrt{r} - 8\sqrt{r} - 8 = \boxed{-16}$$

$$f(f(a)) = a \Rightarrow f(a) = \frac{a+1}{a-1}$$

$$\frac{\frac{a+1}{a-1}}{\frac{a+1}{a-1} - 1} = a \Rightarrow \frac{\frac{a+1}{a-1}}{\frac{a+1-a+1}{a-1}} = a \Rightarrow \frac{a+1}{2} = a \Rightarrow a = a \rightarrow \text{نقطه یک منخرج صوری اند}$$

$$Df = \mathbb{R} - \{1\}$$

$$f(x) = x^2 - 8x, g(x) = \sqrt{x^2 - 1}, f(g(a)) = -8 \quad a = ? \quad -1$$

$$f(x) = -8 \Rightarrow x^2 - 8x = -8 \Rightarrow x^2 - 8x + 8 = 0 \Rightarrow (x-4)^2 = 0 \Rightarrow x = 4$$

$$g(a) = 4 \Rightarrow \sqrt{a^2 - 1} = 4 \Rightarrow a^2 - 1 = 16 \Rightarrow a^2 = 17$$

$$a = \pm \sqrt{17}$$

$$f(x+2) = x^2 - 8x + 4 \quad 4$$

$$x=1 \mid f(1) = 1 - 8 + 4 = -3$$

$$f(1) \times f(-1) = -3 \times 12 = \boxed{-36}$$

$$x=-1 \mid f(-1) = 1 + 8 + 4 = 13$$

$$f(r \cos x + d \cos x) = \tan^2 x \quad -10$$

$$f(r, d) = ?$$

$$\cos x = \frac{1}{r} \Rightarrow \left(r \times \left(\frac{1}{r}\right)^2 + d \times \left(\frac{1}{r}\right)\right) = (\tan^2 x)$$

$$\sin^2 + \cos^2 = 1 \Rightarrow \sin^2 = 1 - \frac{1}{r^2} = \frac{r^2 - 1}{r^2} \Rightarrow \tan^2 x = \frac{\frac{r^2 - 1}{r^2}}{\frac{1}{r^2}} = r^2 - 1 \Rightarrow \tan^2 = r^2 - 1$$

$$\boxed{f(r, d) = r^2 - 1}$$