

۱. $f(x) = \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} \Rightarrow \frac{-2x+1}{x(x-1)} \geq 0$

جدول علامت: $\begin{array}{c|c|c|c|c} & -\infty & 0 & 1 & +\infty \\ \hline & + & - & + & - \end{array}$ $\Rightarrow (-\infty, 0) \cup [\frac{1}{2}, +\infty)$

۲. $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{2}{x-1} + \frac{1}{x+2}} \Rightarrow \frac{\frac{x-2}{x(x+1)}}{\frac{2x+1}{(x-1)(x+2)}} = 0$

$\Rightarrow \frac{x-2}{(x-1)(x+2)} = 0 \Rightarrow x-2=0 \Rightarrow x=2$

همچنین: $\frac{x-2}{(x-1)(x+2)} \geq 0 \Rightarrow x=2$

۳. $\sqrt{(\frac{1}{x})^x - 1} (3x - x^2) \geq 0$

$\Rightarrow \begin{cases} \frac{1}{x} \geq 0 \Rightarrow x > 0 \\ 3x - x^2 \geq 0 \Rightarrow x(3-x) \geq 0 \Rightarrow 0 \leq x \leq 3 \end{cases}$

جدول علامت: $\begin{array}{c|c|c|c|c} & -\infty & 0 & 3 & +\infty \\ \hline & - & + & - & + \end{array}$

۴. $\sqrt[3]{x-1} + \sqrt{x+1} = 3$

$\Rightarrow \begin{cases} x-1 \geq 0 \Rightarrow x \geq 1 \\ 3 - \sqrt[3]{x-1} \geq 0 \Rightarrow \sqrt[3]{x-1} \leq 3 \Rightarrow x-1 \leq 27 \Rightarrow x \leq 28 \end{cases}$

$\Rightarrow D_f = [1, 28]$

۵. $x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0$

جدول علامت: $\begin{array}{c|c|c|c|c} & -\infty & -1 & 2 & +\infty \\ \hline & + & - & + & - \end{array}$

۶. $\sqrt{x^2-1} \Rightarrow x^2-1 \geq 0 \Rightarrow x \leq -1 \text{ یا } x \geq 1$

۷. $\sqrt{x^2-1} \Rightarrow x^2-1 \geq 0 \Rightarrow x \leq -1 \text{ یا } x \geq 1$

۸. $\sqrt{3+ax-x^2}$

$\Rightarrow -x^2 - \frac{1}{2}x + 3 = 0$

$x-2 \Rightarrow 2x^2 + x - 6 = 0$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4 \text{ یا } x = 3$

$a+b = 1$

۹. $f(x) = \begin{cases} 2x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$

۱۰. $1f = f(x) = x$

$\Rightarrow \begin{cases} 2x-2 = x \Rightarrow x=2 \\ 2x+3 = x \Rightarrow x=-3 \end{cases}$

جدول علامت: $\begin{array}{c|c|c|c|c} & -\infty & -3 & 2 & +\infty \\ \hline & - & + & - & + \end{array}$

~~اینجا به دلیل محدودیت فضا، برخی از محاسبات حذف شده است.~~

$$\sin \frac{\pi}{4} + \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$f\left(\frac{1}{\sqrt{2}} + 1\right) = \sqrt{\left(\frac{1}{\sqrt{2}} + 1\right)^2 - 2} = \sqrt{\frac{1}{2} + 2\sqrt{\frac{1}{2}} + 1 - 2} = \sqrt{\frac{1}{2} + 2\sqrt{\frac{1}{2}} - 1}$$

$$\Rightarrow \sqrt{\frac{1}{2} + 1 + \sqrt{2} - \sqrt{2} - 1} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$\boxed{-\frac{1}{\sqrt{2}}}$$

$$f(a) = \frac{a+1}{a-1}$$

$$f(f(a)) = a$$

$$\frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a$$

$$\frac{a+1+a-1}{a-1} = \frac{2a}{a-1} = a \Rightarrow a = a$$

$$a \in \mathbb{R} - \{1\}$$

$$f(g(a)) = -a$$

$$g(a) = \sqrt{a^2 - 1}$$

$$\sqrt{a^2 - 1} = 1 \Rightarrow a^2 - 1 = 1 \Rightarrow a^2 = 2 \Rightarrow a = \pm\sqrt{2}$$

$$f(\sqrt{a^2 - 1}) = \frac{a^2 - 1 - \epsilon\sqrt{a^2 - 1} + 1}{a^2 - 1 - \epsilon\sqrt{a^2 - 1} + 1} = 0 \Rightarrow t - \epsilon t + \epsilon = 0 \Rightarrow (t - \epsilon)^2 = 0 \Rightarrow t = \epsilon$$

$$f(x+r) = x^2 - \omega x + 9$$

$$f(1) \Rightarrow \text{if } x=1 \Rightarrow (1)^2 - \omega + 9 = 22 \Rightarrow \omega = 18$$

$$f(-9) \Rightarrow \text{if } x=-9 \Rightarrow (-9)^2 + \omega + 9 = 72 \Rightarrow \omega = 54$$

$$f(\cos^2 x) + \omega \cos x = \frac{1}{\sqrt{2}} \Rightarrow f(\cos^2 x) + \omega \cos x - \frac{1}{\sqrt{2}} = 0$$

$$\cos^2 x + 10 \cos x - \frac{1}{\sqrt{2}} = 0$$

$$\cos^2 x + 10 \cos x - \frac{1}{\sqrt{2}} = 0$$

$$(\cos x + 10)(\cos x - \frac{1}{10}) = 0$$

$$\cos x = \frac{1}{10} \Rightarrow x = \arccos\left(\frac{1}{10}\right)$$

$$\cos x = -\frac{1}{10} \Rightarrow x = \arccos\left(-\frac{1}{10}\right)$$

$$-1 \leq \cos x \leq 1$$

راه حل دیر سوال (۵)

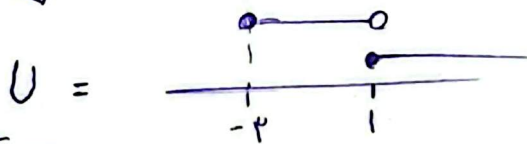
$$g(x) = \sqrt{f(x) - x}$$

$$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$$

if ① $x \geq 1 \Rightarrow g(x) = \sqrt{3x-2-x} \Rightarrow 2x-2 \geq 0 \Rightarrow x \geq 1 \Rightarrow \cap_{1,2} \Rightarrow x \geq 1$

② $x < 1 \Rightarrow g(x) = \sqrt{2x+3-x} \Rightarrow x+3 \geq 0 \Rightarrow x \geq -3 \Rightarrow \cap_{1,2} \Rightarrow [-3, 1)$

$\Downarrow$



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$\Rightarrow [-3, +\infty)$

توجه: تنها به راه حل دوم در این بزرگ تقسیم نشود نه راه حل در بزرگ تلف

راه حل اول هم درست است نه

باجه دل تعیین علامت اگر بفواهی تعیین علامت کن:

x	-3	1	
$3x-2$	shaded	shaded	+
$x+3$	-	+	shaded
f	-	+	+