

الف) $\sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2-1-2x}{x^2-x} \geq 0 \rightarrow \frac{1-2x}{x^2-x} \geq 0 \rightarrow 1-2x \leq 0 \rightarrow x \geq \frac{1}{2}$
 $x^2-x \geq 0 \rightarrow x(x-1) \geq 0$
 $x \leq 0$ یا $x \geq 1$

$D_f: (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $\frac{1}{x+1} - \frac{1}{x} \rightarrow \frac{x - (x+1)}{x(x+1)} = \frac{-1}{x(x+1)} \geq 0$

$\frac{1}{x+1} - \frac{1}{x} \geq 0 \rightarrow \frac{x - (x+1)}{x(x+1)} \geq 0 \rightarrow \frac{-1}{x(x+1)} \geq 0 \rightarrow x(x+1) < 0$
 $x \in (-1, 0)$

$D_f: \mathbb{R} - \{0, -1, \frac{1}{2}, 1\}$

الف) $\sqrt{\left(\frac{1}{x}\right)^n - 1} (3x - 5^n) \rightarrow \left(\frac{1}{x}\right)^n - 1 = 0 \rightarrow \left(\frac{1}{x}\right)^n = 1 \rightarrow x = \pm 1$
 $3x - 5^n = 0 \rightarrow x = \frac{5^n}{3} \rightarrow x \in \mathbb{R}$

$D_f: (-\infty, -1] \cup [1, \infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow y+1 = 9 - 6\sqrt{x-1} + x-1 \rightarrow y = x - 6\sqrt{x-1} + 8$
 $x \geq 1 \rightarrow y \geq -1$
 $\sqrt{x-1} - 6\sqrt{x-1} + 9 \geq 0 \rightarrow (3 - 5\sqrt{x-1})^2 \geq 0$
 $D_f: [1, \infty)$

الف) $\log_x(x^2 - x - 2) \rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x < -1$ یا $x > 2$
 $x^2 - x - 2 \geq 0 \rightarrow x \leq -1$ یا $x \geq 2$
 $D_f: (-\infty, -1) \cup (2, \infty)$

$x = -2 \rightarrow 3 - 2a - 5 \geq -2a - 1 = 0 \rightarrow a = \frac{1}{2}$
 $-2 \times b = P = \frac{c}{a} = -3 \rightarrow b = \frac{3}{2}$
 $a+b = \frac{1}{2} + \frac{3}{2} = 2$

$\sqrt{f(x)} - x \rightarrow f(x) - x \geq 0 \rightarrow f(x) \geq x$
 $x \geq 1 \rightarrow 3x - 2 \geq x \rightarrow 2x \geq 2 \rightarrow x \geq 1$
 $x < 1 \rightarrow 3x, 3 \geq x \rightarrow x \geq -3$
 $D_f: [-3, \infty)$

$$\sin \frac{\pi}{r} = \frac{\sqrt{r}}{r}$$

$$\cos \frac{\pi}{r} = 1$$

$$f(\sin \frac{\pi}{r} + \cos \frac{\pi}{r}) = f(1, \frac{\sqrt{r}}{r}) = r(1 + \frac{\sqrt{r}}{r})^r - r(1 - \frac{\sqrt{r}}{r})^r$$

$$1 - \frac{r}{r} + \sqrt{r}$$

$$r + \frac{r}{r} + \sqrt{r} - r - \sqrt{r} = -\frac{1}{r}$$

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$$f(a) = \frac{a+1}{a-1} \rightarrow f\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a$$

$$f(f(a)) = a \rightarrow a \neq 1 \rightarrow D_f: \mathbb{R} \setminus \{1\}$$

$$\frac{a+1}{a-1} - 1 \neq 0 \rightarrow a+1 \neq a-1 \rightarrow 1 \neq -1 \in \mathbb{R}$$

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$$a^r - r a = -r \rightarrow a^r - r a + r = 0 \rightarrow (a-r)^r = 0 \rightarrow a = r$$

$$g(a) = r \rightarrow \sqrt{a^r - 1} = r \rightarrow a^r - 1 = r^2 \rightarrow a^r = 1 + r^2 \rightarrow a = \pm \sqrt[r]{1+r^2}$$

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$$\begin{aligned} x=1 &\rightarrow f(x) = 1 - \omega + y - r \\ x=-y &\rightarrow f(-y) = ry + y_0 + y = ry \end{aligned} \left\} f(x) + f(-y) = 1 + ry$$

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$$r(\cos^r x + \omega \cos x) = r, \omega \rightarrow r(\cos^r x + \omega \cos x - r, \omega) = 0 \rightarrow \cos x \rightarrow -1 \leq \cos x \leq 1$$

$$\cos x = \frac{-\omega \pm \sqrt{\omega^2 + \omega^2}}{r} = \frac{-\omega \pm \omega}{r} \rightarrow \frac{-1}{r} \propto$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$f(r, \omega) = \frac{(\tan \frac{\omega}{r})^r}{\sqrt{r}}$$

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