

۲۰. آفرین

a) $f(a) = \sqrt{\frac{a-1}{a} - \frac{a}{a-1}} \Rightarrow \frac{a-1}{a} > \frac{a}{a-1} \Rightarrow \frac{a \neq 1, 0}{a \neq 1, 0} \Rightarrow \frac{a-1}{a} > \frac{a}{a-1}$
 $\Rightarrow \frac{a-1}{a} - \frac{a}{a-1} > 0 \Rightarrow \frac{a^2 - 1}{a(a-1)} > 0 \Rightarrow \frac{(a-1)(a+1)}{a(a-1)} > 0 \Rightarrow \frac{a+1}{a} > 0 \Rightarrow a > 0$
 $\Rightarrow D_f = \{x \in \mathbb{R} \mid x > 0, x \neq 1\}$
 b) $f(x) = \frac{1}{x-1} - \frac{1}{x}$
 $\Rightarrow \frac{1}{x-1} - \frac{1}{x} = 0 \Rightarrow \frac{x - (x-1)}{x(x-1)} = 0 \Rightarrow \frac{1}{x(x-1)} = 0 \Rightarrow x(x-1) \neq 0 \Rightarrow x \neq 0, 1$
 $\Rightarrow D_f = \{x \in \mathbb{R} \mid x \neq 0, 1\}$

ا) $f(x) = \sqrt{\left(\frac{1}{x}\right)^n - 9} (x - x^n)$ $O_f =]R - (x, x^n)$
 ب) $\sqrt[n]{n-1} + \sqrt{y+1} = x \Rightarrow \sqrt[n]{n-1} = x - \sqrt{y+1}$
 $\Rightarrow \sqrt[n]{n-1} \leq x \Rightarrow n-1 \leq x^n \Rightarrow n \leq x^n + 1 \Rightarrow O_f = [1, x^n + 1]$

$f(x) = \frac{\log_e (a^2 \cdot x - 2)}{\sqrt{a^2 - 1}}$

$\Rightarrow D_f = (-\infty, -1) \cup \text{[red circle]} (1, \infty)$

$$f(x) = \sqrt{4 + 9x - x^2} \Rightarrow D_f = [-2, 6] \xrightarrow{x=-2} f(x) = 0$$

$\Rightarrow 3 - 2a - 1 = 0 \Rightarrow 2a = -1 \Rightarrow a = -\frac{1}{2}$
 $a = -\frac{1}{2} \rightarrow f(a) = 0 = 2 \cdot \frac{1}{4} - 6 \cdot \frac{1}{2} + 3 = 0 \Rightarrow \frac{1}{2} \pm \sqrt{\frac{1}{4} + 12}$
 $b = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 12}$
 $a + b = -\frac{1}{2} + \frac{1}{2} = 0$
 $b = 2 \pm \sqrt{12}$

$$f(x) = \begin{cases} \mu x - \nu & x \geq 1 \\ \nu x + \mu & x < 1 \end{cases} \quad g(x) = \sqrt{f(x) - a_1}$$

$x \geq 1 \Rightarrow \mu x - \nu - a_1 \geq 0 \Rightarrow \mu x - \nu \geq a_1 \Rightarrow x \geq 1$
 $x < 1 \Rightarrow \nu x + \mu - a_1 \geq 0 \Rightarrow \nu x + \mu \geq a_1 \Rightarrow x \geq \frac{a_1 - \mu}{\nu}$
 $\Rightarrow D_f = \left[\frac{a_1 - \mu}{\nu}, +\infty \right)$

$$f(n) = \sqrt{n^2 - 1} \quad f\left(\sin \frac{\pi}{2} + i \cdot \sin \frac{\pi}{2}\right) = ?$$

$$\sin \frac{\pi}{2} + i \cdot \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(1+i) \Rightarrow f\left(\frac{\sqrt{2}}{2}(1+i)\right)$$

$$\Rightarrow f\left(\frac{\sqrt{2}}{2}(1+i)\right) = \sqrt{\left(\frac{\sqrt{2}}{2}(1+i)\right)^2 - 1} = \sqrt{\frac{2}{4}(1+i)^2 - 1} = \sqrt{\frac{1}{2}(1+i)^2 - 1} = \sqrt{\frac{1}{2}(1+2i-1) - 1} = \sqrt{i - 1} = -\frac{1}{\sqrt{2}}$$

$$f(a) = \frac{a+1}{a-1} \quad f(f(a)) = a \Rightarrow f\left(\frac{a+1}{a-1}\right) = a$$

$$\Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \Rightarrow \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = a \Rightarrow \frac{2a}{2} = a \Rightarrow \frac{2a^2 - 2a}{2a - 2} = a \Rightarrow \frac{a(a-1)}{a-1} = a$$

$a \in \mathbb{R} - \{1\}$

$$f(n) = n^2 - 1 \quad g(n) = \sqrt{n^2 - 1} \quad f(g(a)) = -1$$

$$\Rightarrow f(\sqrt{a^2 - 1}) = -1 \Rightarrow a^2 - 1 - f(\sqrt{a^2 - 1}) = -1$$

$$\Rightarrow a^2 - 1 - \sqrt{a^2 - 1} = -1 \Rightarrow a^2 - \sqrt{a^2 - 1} = 0 \Rightarrow a^2 = \sqrt{a^2 - 1} \Rightarrow a^4 = a^2 - 1 \Rightarrow a^4 - a^2 + 1 = 0$$

$$\leftarrow \frac{a^2 = t}{\Rightarrow a^2 = a} \Rightarrow (t^2 - 1)t + 1 = 0 \Rightarrow (t-1)(t+1)t = 0 \Rightarrow t = 0 \Rightarrow a = \pm \sqrt{0}$$

$$f(n, r) = n^2 - 1 \quad n, r \in \mathbb{N} \Rightarrow f(r) = ?$$

$$n + r = 1 \Rightarrow n = 1 \Rightarrow f(r) = r$$

$$f(-r) \Rightarrow n + r = -r \Rightarrow n = -2r \Rightarrow f(-r) = (-2r)^2 - 1 = 4r^2 - 1$$

$$f(r) \times f(-r) = r \times 4r^2 = 4r^3$$

$$f\left(\frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha\right) = \tan \alpha \quad \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha = \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha = \frac{1}{2} \Rightarrow \cos \alpha + \sqrt{3} \sin \alpha = 1 \Rightarrow \cos \alpha = 1 - \sqrt{3} \sin \alpha$$

$$\Rightarrow \cos \alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3} \Rightarrow \tan \alpha = \frac{1}{\sqrt{3}} \Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{3}}$$