

$$a) f(a) = \sqrt{\frac{a-1}{a} - \frac{a}{a-1}} \Rightarrow \frac{a-1}{a} > \frac{a}{a-1} \quad \frac{a \neq 1, 0}{\frac{a-1}{a} > \frac{a}{a-1}} \Rightarrow \frac{a-1}{a} > \frac{a}{a-1}$$

$$+ \frac{1}{a} - \frac{1}{a-1} = 0 \quad D_f = (-\infty, 0) \cup [1, 1)$$

$$b) f(a) = \frac{1}{a-1} - \frac{1}{a} \quad a \neq 0, -1, -\frac{1}{2}, 1$$

$$\frac{1}{a-1} + \frac{1}{a+1} \neq 0 \Rightarrow \frac{1}{a-1} \neq -\frac{1}{a+1} \Rightarrow \frac{a-1}{a+1} \neq 0 \Rightarrow a \neq -1, \frac{a}{a+1} \neq 0 \Rightarrow a \neq 1, \frac{a}{a+1}$$

ب) $\sqrt[n]{n-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt[n]{n-1} = 3 - \sqrt{y+1}$
 $\Rightarrow \sqrt[n]{n-1} \leq 3 \Rightarrow n-1 \leq 27 \Rightarrow n \leq 28 \Rightarrow O_f = [1, 28]$

$$f(x) = \frac{\log_e(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$$

$$\Rightarrow D_f = (-\infty, -1) \cup \cancel{\mathbb{R}}(1, \infty)$$

$$f(a_1) = \sqrt{1 + a_1 - a_1^2} \Rightarrow D_f = [-1, 6] \xrightarrow{a=-1} f(a) = 0$$

$\Rightarrow 3 - 2a - 1 = 0 \Rightarrow 2a = -1 \Rightarrow a = -\frac{1}{2}$
 $a = -\frac{1}{2} \rightarrow f(a_1) = 0 \Rightarrow 2x^2 - 6x - 6 + 3 = 0 \Rightarrow x = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 12}$
 $\rightarrow b = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 12}$
 $\rightarrow a + b = -\frac{1}{2} + \frac{1}{2} = 0$
 $\rightarrow b = 2x$
 ضرب هر یک از اینها در 3 و با هم جمع کنیم

$$f(x) = \begin{cases} 4x-1 & x \geq 1 \\ 4x+1 & x < 1 \end{cases} \quad g(x) = \sqrt{f(x)-1}$$

$$f(n) = \sqrt{n^2 - 1} \quad f\left(\sin \frac{\pi}{r} + i \cdot \cos \frac{\pi}{r}\right) = ?$$

$$\sin \frac{\pi}{r} + i \cdot \cos \frac{\pi}{r} = \frac{\sqrt{r}}{r} + i \cdot \frac{\sqrt{r}}{r} = \frac{\sqrt{r}}{r} (1 + i) \Rightarrow f\left(\frac{\sqrt{r}}{r} (1 + i)\right)$$

$$\Rightarrow f\left(\frac{\sqrt{r} + i}{r}\right) = \sqrt{\left(\frac{\sqrt{r} + i}{r}\right)^2 - 1} = \sqrt{\frac{r + 2i\sqrt{r} - 1}{r^2} - 1} = \frac{\sqrt{r} + i}{r} - \frac{r}{r} = -\frac{1}{r}$$

$$f(a) = \frac{a+1}{a-1} \quad f(f(a)) = a \Rightarrow f\left(\frac{a+1}{a-1}\right) = a$$

$$\Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \Rightarrow \frac{a(a-1)}{a-1} = a \Rightarrow a \in \mathbb{R} - \{1\}$$

$$f(n) = n^2 - 1 \quad g(n) = \sqrt{n^2 - 1} \quad f(g(a)) = -1$$

$$\Rightarrow f(\sqrt{a^2 - 1}) = -1 \Rightarrow a^2 - 1 - f(\sqrt{a^2 - 1}) = -1 \Rightarrow a^2 - 1 - \sqrt{a^2 - 1} = -1 \Rightarrow a^2 - \sqrt{a^2 - 1} = 0 \Rightarrow a^2 = \sqrt{a^2 - 1} \Rightarrow a^4 = a^2 - 1 \Rightarrow a^4 - a^2 + 1 = 0 \Rightarrow a^2 = \frac{1 \pm \sqrt{1 - 4}}{2} = \frac{1 \pm i\sqrt{3}}{2} \Rightarrow a = \pm \sqrt{\frac{1 \pm i\sqrt{3}}{2}}$$

$$f(n, r) = n^2 - 1 \quad f(r) = ? \quad n + r = r \Rightarrow n = 1 \Rightarrow f(r) = r$$

$$f(-r) \Rightarrow n + r = -r \Rightarrow n = -2r \Rightarrow f(-r) = (-2r)^2 - 1 = 4r^2 - 1$$

$$f(r) \times f(-r) = r \times (4r^2 - 1) = 4r^3 - r$$

$$f\left(\frac{1}{r} \cos \theta + i \sin \theta\right) = \tan \theta \quad f\left(\frac{1}{r} \cos \theta + i \sin \theta\right) = \frac{1}{r} \cos \theta + i \sin \theta$$

$$\Rightarrow \left(\frac{1}{r} \cos \theta + i \sin \theta\right) \cdot \left(\frac{1}{r} \cos \theta + i \sin \theta\right) = \frac{1}{r} \cos \theta + i \sin \theta \Rightarrow \left(\frac{1}{r} \cos \theta + i \sin \theta\right) \cdot \left(\frac{1}{r} \cos \theta + i \sin \theta\right) = \frac{1}{r} \cos \theta + i \sin \theta$$

$$\Rightarrow \left(\frac{1}{r} \cos \theta + i \sin \theta\right) \cdot \left(\frac{1}{r} \cos \theta + i \sin \theta\right) = \frac{1}{r} \cos \theta + i \sin \theta \Rightarrow \left(\frac{1}{r} \cos \theta + i \sin \theta\right) \cdot \left(\frac{1}{r} \cos \theta + i \sin \theta\right) = \frac{1}{r} \cos \theta + i \sin \theta$$

$$\Rightarrow f\left(\frac{1}{r} \cos \theta + i \sin \theta\right) = \frac{1}{r} \cos \theta + i \sin \theta$$