

نام و نام خانوادگی: پرنام پاسخنامه تشریحی تکلیف شماره 5 کلاس: یازم

$$1) f(x) = \sqrt{\frac{x-1}{2x}} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - 2x^2}{x(x-1)} \geq 0 \Rightarrow \frac{-2x^2 + 2x - 1}{x(x-1)} \geq 0$$

$$\frac{0}{+} \frac{-}{+} \frac{1}{+} \Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$$

$$2) f(x) = \frac{1}{x+1} - \frac{2}{x-1} \geq 0 \Rightarrow \frac{3}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow \frac{3(x+2) + 1(x-1)}{(x-1)(x+2)} \neq 0 \Rightarrow 3x+6x-1 \neq 0$$

$$4x+5 \neq 0 \Rightarrow x \neq -\frac{5}{4} \Rightarrow D_f = \mathbb{R} - \{0, -1, -2, -\frac{5}{4}\}$$

$$3) f(x) = \sqrt{(\frac{1}{3}x - 9)(32 - 4x)} \geq 0 \Rightarrow (\frac{1}{3}x - 9)(32 - 4x) \geq 0 \Rightarrow (\frac{1}{3}x - 9) \geq 0 \Rightarrow x \geq 27$$

$$32 - 4x \geq 0 \Rightarrow 32 \geq 4x \Rightarrow 8 \geq x \Rightarrow x \leq 8$$

$$\Rightarrow D_f = (-\infty, 8] \cup [27, +\infty)$$

$$4) \sqrt{4x-1} + \sqrt{4x+1} = 3 \Rightarrow 4x-1 \geq 0 \Rightarrow x \geq \frac{1}{4}, 4x+1 \geq 0 \Rightarrow x \geq -\frac{1}{4}$$

$$3 - \sqrt{4x-1} \geq 0 \Rightarrow 3 \geq \sqrt{4x-1} \Rightarrow 9 \geq 4x-1 \Rightarrow 8 \geq 4x \Rightarrow 2 \geq x$$

$$\Rightarrow D_f = [\frac{1}{4}, 2]$$

$$f(x) = \frac{\log_4(x^2 - 2x - 2)}{\sqrt{x^2 - 1} + 1} \geq 0 \Rightarrow \frac{(x+1)(x-2)}{2} \geq 0 \Rightarrow D_1 = (-\infty, -1) \cup (2, +\infty)$$

$$\sqrt{x^2 - 1} + 1 \geq 0 \Rightarrow x^2 - 1 \geq 0 \Rightarrow (x-1)(x+1) \geq 0 \Rightarrow D_2 = (-\infty, -1] \cup [1, +\infty)$$

$$\Rightarrow D_f = D_1 \cap D_2 = (-\infty, -1) \cup (2, +\infty)$$

$$f(x) = \sqrt{3 + ax - x^2} \Rightarrow \sqrt{3 - 2ax + 4} = 0 \Rightarrow -1 - 2a = 0 \Rightarrow 2a = -1 \Rightarrow a = -\frac{1}{2}$$

$$P = \frac{c}{a} \Rightarrow x_1 \cdot x_2 = \frac{3}{-1} = -3 \Rightarrow b = -3 \Rightarrow b = \frac{3}{2}$$

$$Ax^2 + Bx + C = x^2 - \frac{1}{2}x + 3$$

$$\Rightarrow a+b = \frac{3}{2} - \frac{1}{2} = 1$$

$$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases} \quad g(x) = \sqrt{f(x) - x} \Rightarrow f(x) - x \geq 0 \Rightarrow f(x) \geq x$$

$$\Rightarrow x \geq 1 \Rightarrow f(x) = 3x-2 \Rightarrow 3x-2 \geq x \Rightarrow 2x \geq 2 \Rightarrow x \geq 1 \Rightarrow D_1 = [1, +\infty)$$

$$x < 1 \Rightarrow f(x) = 2x+3 \Rightarrow 2x+3 \geq x \Rightarrow x \geq -3 \Rightarrow D_2 = [-3, +\infty)$$

$$\Rightarrow D_1 \cup D_2 = [-3, +\infty)$$

$$f(n) = 2n^2 - 4n \Rightarrow f\left(\frac{\sqrt{3}}{2} + 1\right) = 2\left(\frac{\sqrt{3}}{2} + 1\right)^2 - 4\left(\frac{\sqrt{3}}{2} + 1\right)$$

$$2\left(\frac{\sqrt{3}}{2} + 1\right)^2 = \frac{3}{4} + 1 + \sqrt{3} = \left(\frac{7}{4} + \sqrt{3}\right) \Rightarrow 3.5 + 2\sqrt{3}$$

$$-4\left(\frac{\sqrt{3}}{2} + 1\right) = -2\sqrt{3} - 4 \Rightarrow \frac{7}{2} + 2\sqrt{3} - 2\sqrt{3} - 4 = \boxed{-\frac{1}{2}}$$

$$f\left(\sin \frac{\pi}{3} + \cos 2\pi\right) = ?$$

$$\frac{\sqrt{3}}{2} + 1 \Rightarrow n = \frac{\sqrt{3}}{2} + 1$$

$$f(n) = \frac{n+1}{n-1} \Rightarrow f(f(n)) = \frac{f(n)+1}{f(n)-1}$$

$$= \frac{\frac{n+1}{n-1} + 1}{\frac{n+1}{n-1} - 1} = \frac{\frac{2n}{n-1}}{\frac{2}{n-1}} = n$$

$$f(f(a)) \rightarrow a \neq 1 \text{ باید باشد}$$

$$f(1) = 1 \Rightarrow \frac{2}{0} = 1 \propto$$

معادله به ازای تمام اعداد حقیقی به جز $a=1$ صحیح است.

$$f(n) = n^2 - 4n, g(n) = \sqrt{n^2 - 1} \text{ و } f(g(a)) = -4 \quad a = ?$$

$$f(t) = t^2 - 4t = -4 \Rightarrow t^2 - 4t + 4 = 0 \Rightarrow (t-2)^2 = 0 \Rightarrow t = 2 \Rightarrow g(a) = 2$$

$$\Rightarrow g(a) = 2 \Rightarrow \sqrt{a^2 - 1} = 2 \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow a = \pm\sqrt{5}$$

$$\boxed{a = \pm\sqrt{5}}$$

$$f(n+3) = n^2 - 5n + 6 \quad f(4) \text{ و } f(-3) = ?$$

$$\left. \begin{aligned} f(4) &= 1 - 5 + 6 = 2 \\ f(-3) &= 36 + 30 + 6 = 72 \end{aligned} \right\} 2 \times 72 = \boxed{144}$$

$$n+3=4 \Rightarrow n=1 \quad n+3=-3 \Rightarrow n=-6$$

$$f(4\cos^2 n + 5\cos n) = \tan^2 n$$

$$f(3,5) = ?$$

$$\cos n = \frac{1}{2} \Rightarrow \left(4\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right)\right) = (3,5) \Rightarrow n = 60^\circ$$

$$f(3,5) = \tan^2 60^\circ \Rightarrow f(3,5) = \sqrt{3}^2 = \boxed{3}$$