

نام و نام خانوادگی علی جمشیدیان پاسخنامه تشریحی تکلیف شماره ۵۵ کلاس B پسر

۲۰. آزمون های نوشتاری

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}}$ $\begin{matrix} x \neq -1 \\ x \neq 0 \\ x \neq 1 \\ x \neq -2 \end{matrix}$

$x \neq 0, x \neq 1$ $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{1-2x}{x(x-1)} \geq 0$

x	$-\infty$	0	$\frac{1}{2}$	1	$+\infty$
$1-2x$	+	+	-	-	+
$x(x-1)$	+	-	-	+	+
$\frac{1-2x}{x(x-1)}$	+	-	+	-	+

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

$\frac{3x+5+x-1}{(x-1)(x+2)} \neq 0 \Rightarrow 4x+4 \neq 0 \Rightarrow x \neq -\frac{0}{4}$ $D = \mathbb{R} - \{-2, -\frac{0}{4}, -1, 0, 1\}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{3}\right)^x - 9\right)(32 - 4^x)}$

$(3^{-x} - 9)(32 - 4^x) \geq 0$
 $\begin{matrix} \searrow & \swarrow \\ x = -2 & x = \frac{5}{2} \end{matrix}$

$\frac{-2}{+} \quad \frac{0}{-} \quad \frac{5}{+}$

$D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3$ $\sqrt{y+1} = 3 - \sqrt[4]{x-1}$
 همیشه بزرگتر از ۰

$3 - \sqrt[4]{x-1} \geq 0 \Rightarrow \sqrt[4]{x-1} \leq 3 \Rightarrow x-1 \leq 81 \Rightarrow x \leq 82$
 $\sqrt[4]{x-1} \geq 0 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1$

$\Rightarrow D_f = [1, 82]$

$f(x) = \frac{\log_7(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$
 همواره مثبت

$x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow x = (-\infty, -1) \cup (2, +\infty)$
 $x^2 - 1 \geq 0 \Rightarrow (x-1)(x+1) \geq 0 \Rightarrow x = (-\infty, -1] \cup [1, +\infty)$

$D_f = ((-\infty, -1) \cup (2, +\infty)) \cap ((-\infty, -1] \cup [1, +\infty)) = [-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{3+ax-x^2}$ $D_f = [-2, b]$ $-x^2+ax+3 \geq 0 \Rightarrow x^2-ax-3 \leq 0$

$-2 \times b = \frac{c}{a} \Rightarrow -2b = -3 \Rightarrow b = \frac{3}{2}$ $9 - 2 + b = -\frac{b}{a} \Rightarrow -2 + b = a \Rightarrow a = -\frac{1}{2}$

$\Rightarrow a+b = -\frac{1}{2} + \frac{3}{2} = 1$

$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$ $f(x) - x \geq 0 \Rightarrow f(x) \geq x \Rightarrow \begin{cases} 3x-2 \geq x & x \geq 1 \\ 2x+3 \geq x & x < 1 \end{cases}$

$\begin{cases} 2x \geq 2 & x \geq 1 \\ x \geq -3 & x < 1 \end{cases}$

$\downarrow$
 $[-3, +\infty) \cap (-\infty, 1) = [-3, 1)$

$D_g = [-3, 1) \cup [1, +\infty) = [-3, +\infty)$

$$f(x) = 2x^2 - 4x \quad f\left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) = 2\left(1 + \frac{\sqrt{2}}{2} + \sqrt{2}\right) - 4\sqrt{2} - 4$$

$$f\left(\frac{\sqrt{2}}{2} + 1\right) = -2 + \frac{\sqrt{2}}{2} = -\frac{1}{2} \quad \text{D}$$

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$$f(x) = \frac{x+1}{x-1} \quad f(f(a)) = a \quad f(f(x)) = \frac{f(x)+1}{f(x)-1} = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} = \frac{x+1}{x-1} = x$$

$$a \in D_f \rightarrow D_f = \mathbb{R} - \{1\} \Rightarrow a \in \mathbb{R} - \{1\} \quad \text{D}$$

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$$f(g(a)) = (g(a))^2 - 4g(a) - 4 \quad (g(a))^2 - 4g(a) + 4 = 0 \Rightarrow (g(a) - 2)^2 = 0$$

$$g(a) = 2$$

$$g(x) \rightarrow 2 = \sqrt{a^2 - 1} \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow a = \pm\sqrt{5} \quad \text{D}$$

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$$f(-2) = (-2)^2 - 2(-2) + 4 = 4 + 4 + 4 = 12$$

$$f(1) = 1 - 2(1) + 4 = 3$$

$$f(-2) \times f(1) = 12 \times 3 = 36 \quad \text{D}$$

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$$4\cos^2 x + 2\cos x = \frac{1}{2} \Rightarrow 4\cos^2 x + 1\cos x - \frac{1}{2} = 0$$

$$(4\cos x - 1)(\cos x + \frac{1}{2}) = 0$$

$$\tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1 - (\frac{1}{4})^2}{(\frac{1}{4})^2} = \frac{\frac{15}{16}}{\frac{1}{16}} = 15 \quad \text{D}$$

$$\cos x = \frac{1}{4} \quad \cos x = -\frac{1}{2}$$

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