

نام و نام خانوادگی علی حبیبیان پاسخنامه تشریحی تکلیف شماره ۵۵ کلاس پسر B

$$\text{الف)} f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$$

$$x \neq 0, x \neq 1 \quad \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{1-x^2}{x(x-1)} \geq 0$$

x	0	$\frac{1}{x}$	1	
$1-x^2$	+	+	-	-
$x(x-1)$	+	-	-	+
$\frac{1-x^2}{x(x-1)}$	+	-	+	-

$$D_f = (-\infty, 0) \cup \left[\frac{1}{x}, 1\right)$$

$$\text{ب)} f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}} \quad \begin{matrix} x \neq -1 \\ x \neq 0 \\ x \neq 1 \\ x \neq -2 \end{matrix}$$

$$\frac{3x+5+x-1}{(x-1)(x+2)} \neq 0 \Rightarrow 4x+4 \neq 0 \Rightarrow x \neq -\frac{0}{4} \quad D_f = \mathbb{R} - \left\{-2, -\frac{0}{4}, -1, 0, 1\right\}$$

$$\text{الف)} f(x) = \sqrt{\left(\left(\frac{1}{3}\right)^x - 9\right)(32 - 4^x)}$$

$$\left(3^{-x} - 9\right)(32 - 4^x) \geq 0$$

$$\frac{-2}{+} \quad \frac{0}{-} \quad \frac{5}{+}$$

$$D_f = (-\infty, 2] \cup \left[\frac{5}{2}, +\infty\right)$$

$$\text{ب)} \sqrt[3]{x-1} + \sqrt{y+1} = 3 \quad \sqrt[3]{y+1} = 3 - \sqrt[3]{x-1}$$

$$3 - \sqrt[3]{x-1} \geq 0$$

$$3 \geq \sqrt[3]{x-1} \Rightarrow x-1 \leq 1 \quad x \leq 2$$

$$\Rightarrow D_f = [1, 2]$$

$$f(x) = \frac{\log_2(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$$

$$x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow x = (-\infty, -1) \cup (2, +\infty)$$

$$x^2 - 1 \geq 0 \Rightarrow (x-1)(x+1) \geq 0 \Rightarrow x = (-\infty, -1] \cup [1, +\infty)$$

$$D_f = ((-\infty, -1) \cup (2, +\infty)) \cap ((-\infty, -1] \cup [1, +\infty)) = [-\infty, -1) \cup (2, +\infty)$$

$$f(x) = \sqrt{3+ax-x^2} \quad D_f = [-2, b] \quad -x^2+ax+3 \geq 0 \Rightarrow x^2-ax-3 \leq 0$$

$$-2 \times b = \frac{c}{a} \Rightarrow -2b = -3 \Rightarrow b = \frac{3}{2} \quad 9 - 2 + b = -\frac{b}{a} \Rightarrow -2 + b = a \Rightarrow a = -\frac{1}{2}$$

$$\Rightarrow a+b = -\frac{1}{2} + \frac{3}{2} = 1$$

$$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases} \quad f(x) - x \geq 0 \Rightarrow f(x) \geq x \Rightarrow \begin{cases} 3x-2 \geq x \Rightarrow x \geq 1 \\ 2x+3 \geq x \Rightarrow x \geq -3 \end{cases}$$

$$\downarrow$$

$$D_g = [-3, 1) \cup [1, +\infty) = [-3, +\infty)$$

1

2

3

4

5

$$f(x) = 2x^2 - 4x \quad f\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) = 2\left(1 + \frac{\sqrt{2}}{2} + \sqrt{2}\right) - 4\sqrt{2} - 4$$

$$f\left(\frac{\sqrt{2}}{2} + 1\right) = -2 + \frac{\sqrt{2}}{2} = -\frac{1}{2}$$

6

$$f(x) = \frac{x+1}{x-1} \quad f(f(a)) = a \quad f(f(x)) = \frac{f(x)+1}{f(x)-1} = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} = \frac{x+1}{x-1} = x$$

$$a \in D_f \rightarrow D_f = \mathbb{R} - \{1\} \Rightarrow a \in \mathbb{R} - \{1\}$$

7

$$f(g(a)) = (g(a))^2 - 4g(a) - 4 \quad (g(a))^2 - 4g(a) + 4 = 0 \Rightarrow (g(a) - 2)^2 = 0$$

$$g(a) = 2$$

$$g(x) \rightarrow 2 = \sqrt{a^2 - 1} \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow a = \pm\sqrt{5}$$

8

$$f(-2) = (-2)^2 - 2(-2) + 4 = 4 + 4 + 4 = 12$$

$$f(1) = 1 - 2(1) + 4 = 3$$

$$f(-2) \times f(1) = 12 \times 3 = 36$$

9

$$4 \cos^2 x + 2 \cos x = \frac{1}{2} \Rightarrow 4 \cos^2 x + 1 \cdot \cos x - \frac{1}{2} = 0$$

$$(4 \cos x - 1)(\cos x + \frac{1}{2}) = 0$$

$$\cos x = \frac{1}{4} \quad \cos x = -\frac{1}{2}$$

10

$$\tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1 - \left(\frac{1}{4}\right)^2}{\left(\frac{1}{4}\right)^2} = \frac{\frac{15}{16}}{\frac{1}{16}} = 15$$