

نام و نام خانوادگی: پاسخنانه تشریحی تکلیف شماره ۵... کلاس: یازدهم B

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \begin{cases} x \neq 0 \\ x-1 \neq 0 \rightarrow x \neq 1 \end{cases}$
 $\frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow x^2 - 2x + 1 - x^2 = -2x + 1 \geq 0 \Rightarrow \frac{1-2x}{x(x-1)} \geq 0$

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{x}{x-1}}{\frac{x}{x-1} + \frac{1}{x+1}} \Rightarrow x \neq 0, x-1 \neq 0 \Rightarrow x \neq 1, x+1 \neq 0 \Rightarrow x \neq -1$
 $\frac{1}{x-1} + \frac{1}{x+1} \neq 0 \rightarrow \frac{x+1}{(x-1)(x+1)} + \frac{x-1}{(x-1)(x+1)} \neq 0 \rightarrow \frac{x+1+x-1}{(x-1)(x+1)} \neq 0 \rightarrow \frac{2x}{(x-1)(x+1)} \neq 0$
 $2x \neq 0 \Rightarrow x \neq 0$
 $D_f = \mathbb{R} - \{-1, 0, 1\}$

الف) $f(x) = \sqrt{((\frac{1}{x})^x - 9)(32 - x^x)} \Rightarrow ((\frac{1}{x})^x - 9)(32 - x^x) \geq 0 \rightarrow (\frac{1}{x})^x - 9 = 0 \rightarrow x^{-x} = 9 \rightarrow x^x = \frac{1}{9}$
 $\rightarrow -x = 2 \Rightarrow x = -2$
 $32 - x^x = 0 \rightarrow x^x = 32 \rightarrow (2^5)^x = 2^5 \rightarrow 2^{5x} = 2^5 \rightarrow 5x = 5 \rightarrow x = 1$
 $D_f = (-\infty, -2] \cup [1, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y-1} = 3 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1, y-1 \geq 0 \Rightarrow y \geq 1$

$D_f = \{(x, y) | x \geq 1, y \geq 1\}$
 $\sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow \sqrt{y+1} \leq 3 \rightarrow y+1 \leq 9 \rightarrow y \leq 8$
 $\sqrt{x-1} \geq 0 \rightarrow x-1 \geq 0 \rightarrow x \geq 1$
 $D_f = [1, 9] \times [1, 8]$

$f(x) = \frac{\log_f(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \Rightarrow x^2 - x - 2 \geq 0 \rightarrow (x-2)(x+1) \geq 0 \rightarrow x < -1, x > 2$
 $x^2 - 1 \geq 0 \rightarrow x \leq -1, x \geq 1$
 $\sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \geq 0 \rightarrow \sqrt{x^2 - 1} + 1 \geq 1$
 $D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{3+ax-x^2} \quad D_f = [-2, b]$
 $3+a(-2)-(-2)^2 = 0 \rightarrow 3-2a-4 = 0 \rightarrow -2a-1 = 0 \rightarrow a = -\frac{1}{2}$
 $3 - \frac{1}{2}x - x^2 = 0 \rightarrow 2x^2 + x - 6 = 0 \rightarrow (2x+3)(x-2) = 0$
 $x^2 + \frac{1}{2}x - 3 = 0 \rightarrow \sum_{\text{ریشه}} = -\frac{1}{2} \rightarrow -2+b = -\frac{1}{2} \rightarrow b = \frac{3}{2}$
 $a+b = -\frac{1}{2} + \frac{3}{2} = 1$

$g(x) = \sqrt{f(x)} - x \quad f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$
 $f(x) - x \geq 0 \xrightarrow{x \geq 1} 3x-2-x \geq 0 \rightarrow 2x-2 \geq 0 \rightarrow x \geq 1$
 $x < 1 \rightarrow 2x+3-x \geq 0 \rightarrow x+3 \geq 0 \rightarrow x \geq -3 \xrightarrow{x < 1} -3 \leq x < 1$
 $D_f = [-3, +\infty)$

$f(x) = \sqrt{x} - x$ $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow \sqrt{\frac{2}{4}} + 1 \rightarrow f(1 + \sqrt{\frac{2}{4}})$ $f(\sin \frac{\pi}{4} + \cos \frac{\pi}{4})$ $\sqrt{1 + \sqrt{\frac{2}{4}}} - f(1 + \sqrt{\frac{2}{4}}) \Rightarrow \sqrt{\frac{2}{4} + 1} - (1 + \sqrt{\frac{2}{4}}) = \frac{\sqrt{2}}{2} + 1 - 1 - \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2} - \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$	6
$f(x) = \frac{x+1}{x-1} \quad f(f(a)) = a \quad f(f(x)) = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1}$ $\frac{x+1 + x-1}{x-1} = \frac{2x}{x-1} \quad f(f(x)) = \frac{2x}{x-1} \times \frac{x-1}{2} = x$ $\frac{x+1 - x+1}{x-1} = \frac{2}{x-1} \quad f(f(a)) = a \rightarrow \underline{a \neq 1}$ $\underline{a \in \mathbb{R} - \{1\}}$	7
$f(x) = x^2 - x, \quad g(x) = \sqrt{x-1}, \quad f(g(a)) = -1$ $[g(a)]^2 - f(g(a)) = -1$ $g(a)^2 - f(g(a)) + 1 = 0 \rightarrow (g(a) - 1)^2 = 0 \rightarrow g(a) = 1 \rightarrow \sqrt{a-1} = 1 \xrightarrow{\text{square}}$ $a-1 = 1 \rightarrow a = 2 \rightarrow \underline{a = \pm \sqrt{2}}$	8
$f(x+3) = x^2 - 2x + 4 \rightarrow t = x+3 \rightarrow x = t-3$ $f(t) = (t-3)^2 - 2(t-3) + 4 = t^2 - 6t + 9 - 2t + 6 + 4 = t^2 - 8t + 19$ $f(t) = 14 - 8t + 19 = 2$ $f(-3) = 9 + 24 + 19 = 42$ $f(t)f(-3) = 2 \times 42 = \underline{84}$	9
$f(\cos x + \sin x) = \tan^2 x$ $f(\cos x, \sin x) = ? \rightarrow \cos^2 x + \sin^2 x = 1 \rightarrow t = \cos x \rightarrow t^2 + \sin^2 x = 1 \xrightarrow{\text{square}}$ $t^2 + 1 - t^2 = 1 \rightarrow (t-1)(t+1) = 0 \rightarrow t = 1, t = -1$ $-1 \leq \cos x \leq 1 \rightarrow -1 \leq t \leq 1 \rightarrow \cos x = -1$ $\tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1 - 1}{1} = 0 \rightarrow \underline{f(\cos x, \sin x) = 0}$	10

$$\text{ب-۲)} \quad \sqrt{x-1} + \sqrt{y+1} = 3 \quad , \quad \underbrace{\sqrt{y+1}}_{\text{مقدار نامشخص}} = \underbrace{3 - \sqrt{x-1}}_{\text{مقدار نامشخص!}} \rightarrow 3 - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 3 \rightarrow x \leq 10 \quad \mathbb{I}$$

$$\downarrow$$

$$x-1 \geq 0 \rightarrow x \geq 1 \quad \mathbb{I}$$

$$\mathbb{I}, \mathbb{I} \rightarrow 1 \leq x \leq 10 \rightarrow D_f = [1, 10]$$