

نام و نام خانوادگی: پاسخنانه تشریحی تکلیف شماره ۵... کلاس: یازدهم ریاضی B.....

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \begin{cases} x \neq 0 \\ x-1 \neq 0 \rightarrow x \neq 1 \end{cases}$
 $\frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow x^2 - 2x + 1 - x^2 = -2x + 1 \Rightarrow \frac{1-2x}{x(x-1)} \geq 0$

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+1}} \Rightarrow \begin{cases} x+1 \neq 0 \Rightarrow x \neq -1 \\ x-1 \neq 0 \Rightarrow x \neq 1 \\ x+1 \neq 0 \Rightarrow x \neq -1 \end{cases}$
 $\frac{1}{x-1} + \frac{1}{x+1} \neq 0 \rightarrow \frac{x(x+1) + (x-1)}{(x-1)(x+1)} \neq 0$
 $\frac{x^2 + x + x - 1}{(x-1)(x+1)} \neq 0 \rightarrow \frac{x^2 + 2x - 1}{(x-1)(x+1)} \neq 0$
 $D_f = \mathbb{R} - \{-1, -\frac{1}{2}, 0, 1\}$

الف) $f(x) = \sqrt{((\frac{1}{x})^x - 9)(32 - x^x)} \Rightarrow ((\frac{1}{x})^x - 9)(32 - x^x) \geq 0 \rightarrow (\frac{1}{x})^x - 9 = 0 \rightarrow x^{-x} = 9$
 $\rightarrow -x = 2 \Rightarrow x = -2$
 $32 - x^x = 0 \rightarrow x^x = 32 \rightarrow (2^5)^x = 2^5 \rightarrow 2^{5x} = 2^5 \rightarrow 5x = 5 \rightarrow x = 1$
 $D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y-1} = 3 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1$ و $y-1 \geq 0 \Rightarrow y \geq 1$

$D_f = \{(x, y) | x \geq 1, y \geq 1\} \Leftarrow$ دامنه رابطه

$D_z = [1, +\infty) \Leftarrow$ دامنه ایس

$f(x) = \frac{\log_f(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \Rightarrow x^2 - x - 2 \geq 0 \rightarrow (x-2)(x+1) \geq 0 \rightarrow x < -1$ و $x > 2$
 $x^2 - 1 \geq 0 \rightarrow x \leq -1$ و $x \geq 1$
 $\sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \geq 0 \Rightarrow \sqrt{x^2 - 1} + 1 \geq 1$
 $D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{3+ax-x^2}$ $D_f = [-2, b]$

$3+a(-2)-(-2)^2 = 0 \rightarrow 3-2a-4 = 0 \rightarrow -2a-1 = 0 \rightarrow a = -\frac{1}{2}$

$3 - \frac{1}{2}x - x^2 = 0 \rightarrow 2x^2 + x - 6 = 0 \rightarrow (2x+3)(x-2) = 0$

$x^2 + \frac{1}{2}x - 3 = 0 \rightarrow \sum_{\text{ریشه}} = -\frac{1}{2} \rightarrow -2+b = -\frac{1}{2} \rightarrow b = \frac{3}{2}$

$a+b = -\frac{1}{2} + \frac{3}{2} = 1$

$g(x) = \sqrt{f(x)} - x$ $f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$

$f(x) - x \geq 0 \xrightarrow{x \geq 1} 3x-2-x \geq 0 \rightarrow 2x-2 \geq 0 \rightarrow x \geq 1$

حال $x < 1 \rightarrow 2x+3-x \geq 0 \rightarrow x+3 \geq 0 \rightarrow x \geq -3 \xrightarrow{x < 1} -3 \leq x < 1$

$D_f = [-3, +\infty)$

$f(x) = \sqrt{x} - x$ $\sin \frac{\pi}{p} = \frac{\sqrt{p}}{p} \Rightarrow \sqrt{\frac{p}{p}} + 1 \rightarrow f(1 + \sqrt{\frac{p}{p}})$ $f(\sin \frac{\pi}{p} + \cos \pi) = \cos \pi = 1$ $\sqrt{1 + \sqrt{\frac{p}{p}}} - f(1 + \sqrt{\frac{p}{p}}) \Rightarrow \sqrt{1 + \sqrt{\frac{p}{p}}} - (1 + \sqrt{\frac{p}{p}}) = \sqrt{p} + \sqrt{p} - 1 - \sqrt{p}$ $= \sqrt{p} - 1 \rightarrow \sqrt{p} - 1 = -\frac{1}{p}$	6
$f(x) = \frac{x+1}{x-1} \quad f(f(a)) = a \quad f(f(x)) = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1}$ $\frac{x+1 + x-1}{x-1} = \frac{2x}{x-1} \quad f(f(x)) = \frac{2x}{x-1} \times \frac{x-1}{2} = x$ $\frac{x+1 - x+1}{x-1} = \frac{2}{x-1} \quad f(f(a)) = a \rightarrow \underline{a \neq 1}$ $\underline{a \in \mathbb{R} - \{1\}}$	7
$f(x) = x^2 - x, g(x) = \sqrt{x-1}, f(g(a)) = -1$ $[g(a)]^2 - f(g(a)) = -1$ $g(a)^2 - f(g(a)) + 1 = 0 \rightarrow (g(a) - 1)^2 = 0 \rightarrow g(a) = 1 \rightarrow \sqrt{a^2 - 1} = 1 \xrightarrow{\text{square}}$ $a^2 - 1 = 1 \rightarrow a^2 = 2 \rightarrow \underline{a = \pm \sqrt{2}}$	8
$f(x+3) = x^2 - 2x + 4 \rightarrow t = x+3 \rightarrow x = t-3$ $f(t) \times f(-3) = ?$ $f(t) = (t-3)^2 - 2(t-3) + 4 = t^2 - 6t + 9 - 2t + 6 + 4 = t^2 - 8t + 19$ $f(t) = 14 - 8t + 19 = 3$ $f(-3) = 9 + 24 + 19 = 42$ $f(t)f(-3) = 3 \times 42 = \underline{126}$	9
$f(\cos x + \sin x) = \tan^2 x$ $f(\cos x, \sin x) = ? \rightarrow \cos x + \sin x = \frac{1}{\sqrt{2}} \rightarrow t = \cos x \rightarrow t + \sin x = \frac{1}{\sqrt{2}} \xrightarrow{\text{square}}$ $t^2 + 1 + 2t - 1 = 0 \rightarrow (t+1)(t) = 0 \rightarrow t = -1, t = 0$ $-1 \leq \cos x \leq 1 \rightarrow -\frac{1}{\sqrt{2}} \leq \cos x \leq \frac{1}{\sqrt{2}}$ $\tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1 - \frac{1}{2}}{\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1 \rightarrow \underline{f(\cos x, \sin x) = 1}$	10