

$$f(x) = \frac{1}{x+1} - \frac{2}{x}$$

$$\frac{1}{x+1} + \frac{1}{x+2}$$

$$x+1 \neq 0 \rightarrow x \neq -1$$

$$x-1 \neq 0 \rightarrow x \neq 1$$

$$x+2 \neq 0 \rightarrow x \neq -2$$

$$\frac{1}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{x+2}{(x-1)(x+2)} \neq 0 \rightarrow x \neq -\frac{2}{1}$$

$$D_f = \mathbb{R} - \left\{ -1, 1, -\frac{2}{1} \right\}$$

ب) * مخرج ها نباید مفر شوند.

$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{x^2-2x+1-x^2}{x(x-1)}}$$

$$= \sqrt{\frac{-2x+1}{x(x-1)}} \geq 0 \rightarrow \frac{x}{x(x-1)} \geq 0$$

$$\Rightarrow D_f = (-\infty, 0) \cup \left[\frac{1}{2}, +\infty\right)$$

$$1) \sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow$$

$$\sqrt{y+1} = 3 - \sqrt{x-1} \quad \text{توان ۵}$$

$$y+1 = 9 + x-1 - 6\sqrt{x-1} \rightarrow$$

$$y = 8 + \sqrt{x-1} - 6\sqrt{x-1} \rightarrow x \geq 1$$

$$2) \sqrt{y+1} \geq 0 \rightarrow 3 - \sqrt{x-1} \geq 0$$

$$\sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10$$

$$D_y = [1, 10]$$

$$f(x) = \sqrt{(3^x - 9)(3^x - 2^x)} \geq 0$$

$$\Rightarrow D_f = (-\infty, -2] \cup [2, 5 + \infty)$$

$$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$$

$$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x < -1 \text{ یا } x > 2$$

$$x^2 - 1 \geq 0 \rightarrow x \leq -1 \text{ یا } x \geq 1$$

$$\sqrt{x^2 - 1} \geq 0 \rightarrow \sqrt{x^2 - 1} + 1 \geq 1$$

$$D_f = (-\infty, -1] \cup [2, +\infty)$$

هیچ پایه مفرقی ندارد.

$$f(x) = \sqrt{3+ax-x^2} \rightarrow -x^2+ax+3 \geq 0 \rightarrow \frac{-2}{-1} + \frac{b}{-1} \rightarrow D_f = [-2, b]$$

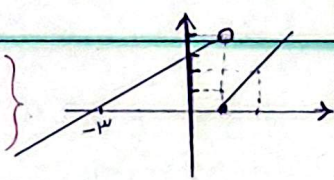
$$-(-2)^2 - 2a + 3 = 0 \rightarrow -2a - 1 = 0 \rightarrow a = -\frac{1}{2}$$

$$\rightarrow x = -2, b = x = \frac{3}{2}$$

$$\Rightarrow a+b = -\frac{1}{2} + \frac{3}{2} = 1$$

$$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases} \rightarrow f(x) - x = \begin{cases} 2x-2 & x \geq 1 \\ x+3 & x < 1 \end{cases}$$

$$f(x) - x \geq 0 \rightarrow \begin{cases} x \geq 1 \rightarrow 2x-2 \geq 0 \rightarrow x \geq 1 \\ x < 1 \rightarrow x+3 \geq 0 \rightarrow x \geq -3 \end{cases} \Rightarrow D_g = [-3, +\infty)$$



$$f(x) = 2x^2 - 4x = 2x(x-2)$$

$$f\left(\sin \frac{\pi}{3} + \cos \frac{\pi}{3}\right) = ? \rightarrow f\left(\frac{2+\sqrt{3}}{2}\right) = ? = \frac{2 \times \frac{2+\sqrt{3}}{2} - 4 \times \frac{2+\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}-4-2\sqrt{3}}{2} = \frac{-2-\sqrt{3}}{2}$$

$$(2+\sqrt{3})\left(\frac{\sqrt{3}-2}{2}\right) = \frac{3-4}{2} = -\frac{1}{2}$$

$$f(x) = \frac{x+1}{x-1} \rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a$$

$$f(f(a)) = a \rightarrow \text{به ازای چه مقادیر } a?$$

به ازای هر مقدار a به جز ریشه های منفرجه

$$a-1 \neq 0 \rightarrow a \neq 1$$

$$\frac{a+1}{a-1} - 1 \neq 0 \rightarrow \frac{2}{a-1} \neq 0 \checkmark$$

$$\Rightarrow D_a = \mathbb{R} - \{1\}$$

$$f(x) = x^2 - 4x$$

$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(a)) = -4 \rightarrow g(a)^2 - 4g(a) = -4 \rightarrow g(a)^2 - 4g(a) + 4 = 0$$

$$(g(a) - 2)^2 = 0 \rightarrow g(a) = 2$$

$$\rightarrow \sqrt{a^2 - 1} = 2 \rightarrow a = \pm \sqrt{5} \checkmark$$

$$f(x+3) = x^2 - 2x + 9$$

$$x=1 \rightarrow f(4) = 1 - 2 + 9 = 8$$

$$x=-4 \rightarrow f(-1) = 16 + 8 + 9 = 33$$

$$\rightarrow f(4) \times f(-1) = 8 \times 33 = 264$$

$$f(4 \cos^2 x + 5 \cos x) = \tan^2 x, \quad f(3, 5) = ?$$

$$4 \cos^2 x + 5 \cos x = 3, 5 \rightarrow 4 \cos^2 x + 5 \cos x - 3 = 0$$

$$\cos^2 x + 5 \cos x - 3 = 0 \rightarrow (\cos x + 3)(\cos x - 1) = 0$$

$$\begin{cases} \cos x = 1 \\ \cos x = -3 \end{cases}$$

$$f(3, 5) = \tan^2 x = \frac{1}{\cos^2 x} - 1 = \frac{1}{1} - 1 = 0$$