

۲. ازین معادله استفاده کنید

$$2f(\omega) = f(\omega) + a \rightarrow 2((\omega+1)(\omega+1)) = 3a - f + a \rightarrow 1f + 1f = 3a - f + a$$

$$\rightarrow 10a = -11 \rightarrow a = \frac{-11}{10} = -1,1 = \frac{-9}{10}$$

$$\rightarrow a = \frac{-9}{10} = -1,1$$

$$f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = \sqrt{2}-\sqrt{3} + \frac{1}{\sqrt{2}-\sqrt{3}} + 2 + \sqrt{2}+\sqrt{3} + \frac{1}{\sqrt{2}+\sqrt{3}} + 2$$

$$\rightarrow (2-\sqrt{3})(2+\sqrt{3}) = 1 \rightarrow \frac{1}{\sqrt{2}-\sqrt{3}} \times \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \frac{\sqrt{2}+\sqrt{3}}{2-3} = -(\sqrt{2}+\sqrt{3})$$

$$\frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{2}-\sqrt{3}}{2-3} = -(\sqrt{2}-\sqrt{3})$$

$$\rightarrow A = \sqrt{2}-\sqrt{3} + \sqrt{2}+\sqrt{3} + 4 = 2\sqrt{2} + 4$$

$$\rightarrow A^2 = 2 + \sqrt{2} + 2 - \sqrt{2} + 2\sqrt{2}(\sqrt{2}+\sqrt{3}) = 4 + 2\sqrt{2} + 2\sqrt{3} + 4 = 8 + 2\sqrt{2} + 2\sqrt{3}$$

$$\rightarrow A^2 = 4 + 2\sqrt{2} + 2\sqrt{3} = 9 \rightarrow A = \sqrt{5} \quad (II)$$

$$\frac{(II)}{A=\sqrt{5}} \rightarrow f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = 2A + 4 = 2\sqrt{5} + 4 \rightarrow f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = 4 + 2\sqrt{5}$$

$$\left\{ \begin{array}{l} 2f(x) - 3f(-x) = 4x^2 - x \xrightarrow{x^2} 4f(x) - 9f(-x) = 16x^2 - 4x \\ 2f(-x) - 3f(x) = 4x^2 + x \xrightarrow{x^2} 4f(-x) - 9f(x) = 16x^2 + 4x \end{array} \right.$$

$$\rightarrow -4f(x) = 16x^2 + x \rightarrow f(x) = -4x^2 - \frac{x}{4}$$

$$\rightarrow f(x) = -4x^2 - \frac{x}{4} = -4x^2 - \frac{x}{4}$$

$$\left. \begin{array}{l} x=0 \rightarrow 2f(0) - 0 = 3m - 1 \rightarrow 2f(0) = 3m - 1 \rightarrow 4f(0) = 6m - 2 \\ x=1 \rightarrow (0 \times f(1)) + 9f(0) = 16 + 3m + 1m - 1 \rightarrow 9f(0) = 16 + 4m \end{array} \right\} \rightarrow 4m - 2 = 6m + 1 \rightarrow 4m - 6m = 1 + 2 \rightarrow -2m = 3 \rightarrow m = -\frac{3}{2}$$

$$m = -\frac{3}{2} \rightarrow 2f(0) = \frac{3 \times -3}{2} - 1 \rightarrow 2f(0) = \frac{-9}{2} - 1 \rightarrow f(0) = \frac{-11}{4}$$

$$\rightarrow f(0) = \frac{-11}{4}$$

$$\left. \begin{array}{l} f(x) = ax + b \\ f\left(\frac{1}{x}\right) = a\left(\frac{1}{x}\right) + b = \frac{a}{x} + b \end{array} \right\} \rightarrow f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b = ax + \frac{a}{x} + 2b$$

$$\frac{3x^2 - 12x + 13}{x^2} \xrightarrow{\text{مخرج مشترک}} \frac{3x^2 - 12x + 13}{x^2} = \frac{3x^2 - 12x + 13}{x^2} = 3 - \frac{12}{x} + \frac{13}{x^2}$$

$$\frac{3x^2 - 12x + 13}{x^2} = 3 - \frac{12}{x} + \frac{13}{x^2} = ax + \frac{a}{x} + 2b$$

$$\left\{ \begin{array}{l} a = 3 \rightarrow a = 3 \\ b = -12 \rightarrow b = -6 \end{array} \right. \rightarrow f(x) = 3x - 6 \rightarrow f\left(\frac{1}{x}\right) = -3 - 6 = -9$$

$$\rightarrow f\left(\frac{1}{x}\right) = -9$$

$$\left. \begin{aligned} f(x) &= (x-1)\sqrt{1-x} \rightarrow 1-x \geq 0 \rightarrow x \leq 1 \rightarrow D_f = (-\infty, 1] \\ g(x) &= -\sqrt{(1-x)^3} \rightarrow (1-x)^3 \geq 0 \rightarrow 1-x \geq 0 \rightarrow x \leq 1 \rightarrow D_g = (-\infty, 1] \end{aligned} \right\} \rightarrow D_f = D_g$$

این دامنه ها برابرند باید مطمئن شویم آیا این دو تابع برابری می کنند

$$g(x) = -\sqrt{(1-x)^3} \xrightarrow{1-x = t} g(x) = -\sqrt{t^3} = -(1-x)\sqrt{1-x} = (x-1)\sqrt{1-x}$$

$$\rightarrow g(x) = f(x) \rightarrow g(x) = f(x) \text{ (دو تابع برابرند)}$$

$$\left. \begin{aligned} f(x) &= x\sqrt{x} \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty) \\ g(x) &= \sqrt{x^3} \rightarrow x^3 \geq 0 \rightarrow x \geq 0 \rightarrow D_g = [0, +\infty) \end{aligned} \right\} \rightarrow D_f = D_g$$

این دو تابع هر دو تابع برابرند و دامنه های آنها یکسان است پس این دو تابع باید یکدیگر برابرند.

$$g(x) = \sqrt{x^3} = \sqrt{x^2 \times x} = \sqrt{x^2} \times \sqrt{x} \xrightarrow{x \geq 0} g(x) = x\sqrt{x}$$

$$\rightarrow g(x) = f(x) \text{ (دو تابع برابرند)}$$

$$\left. \begin{aligned} f(x) &= [\sqrt{x} - \sqrt{x}] \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty) \\ g(x) &= \sqrt{x-|x|} \rightarrow x-|x| \geq 0 \xrightarrow{x \geq 0} g(x) = 0 \end{aligned} \right\} \rightarrow D_g = [0, +\infty)$$

از آنجا که $D_g = D_f$ و هر دو تابع در هر نقطه از دامنه برابرند پس این دو تابع برابرند.

$$\rightarrow D_g = D_f \rightarrow \text{این دو تابع برابرند}$$

$$\rightarrow f(x) = g(x) \text{ (دو تابع برابرند)}$$

برای $x < 0$: $f(x) = [\sqrt{x} - \sqrt{x}] = 0$, $g(x) = \sqrt{x-|x|} = \sqrt{x-x} = 0$

$$a) \left(\frac{f+g}{f-g} \right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} \Rightarrow \left\{ \begin{aligned} f(1) &= 1 \\ g(1) &= 1 \end{aligned} \right\} \rightarrow \frac{1+1}{1-1} = \frac{2}{0} = \text{تعریف نشده است}$$

$$b) f(0)g(-1) = 0 \times 1 = 0$$

$$a) g(x) = f\left(\frac{1}{x}-1\right) \rightarrow \left\{ \begin{aligned} \frac{1}{x}-1 \geq 1 &\rightarrow x \leq \frac{1}{2} \\ \frac{1}{x}-1 \geq -1 &\rightarrow x \geq 0 \\ \frac{1}{x}-1 = 0 &\rightarrow x = 1 \\ \frac{1}{x}-1 = -1 &\rightarrow x = \frac{1}{2} \end{aligned} \right\} \rightarrow g(x) = \left\{ \left(-1, \frac{1}{2}\right), \left(\frac{1}{2}, 1\right), \left(1, 0\right), \left(0, 1\right) \right\}$$

$$b) h(x) = f(|x|) \rightarrow |x| \geq 0 \rightarrow \left\{ \begin{aligned} x=0 &\rightarrow h(0) = f(0) = 0 \\ x=1 &\rightarrow h(1) = f(1) = 1 \\ x=-1 &\rightarrow h(-1) = f(1) = 1 \end{aligned} \right\}$$

$$\rightarrow h(x) = \left\{ (-1, 1), (0, 0), (1, 1) \right\}$$