

$$2f(\omega) = f(\omega) + a \rightarrow 2((\omega+1)(\omega+1)) = 3a - f + a \rightarrow 4\omega + 4 = 3a - f + a$$

$$\rightarrow 4\omega = -1 \rightarrow \omega = \frac{-1}{4} = -1/4 = \frac{9}{\omega}$$

$$\rightarrow a = \frac{9}{\omega} = -1/4$$

$$f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = \sqrt{2}-\sqrt{3} + \frac{1}{\sqrt{2}-\sqrt{3}} + 2 + \sqrt{2}+\sqrt{3} + \frac{1}{\sqrt{2}+\sqrt{3}} + 2$$

$$\rightarrow (\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3}) = 1 \rightarrow \frac{1}{\sqrt{2}-\sqrt{3}} \times \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \frac{\sqrt{2}+\sqrt{3}}{2-3} = -(\sqrt{2}+\sqrt{3})$$

$$\frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{2}-\sqrt{3}}{2-3} = -(\sqrt{2}-\sqrt{3})$$

$$\rightarrow A = \sqrt{2}-\sqrt{3} + \sqrt{2}+\sqrt{3}$$

$$\rightarrow A^2 = 2 + \sqrt{2} + 2 - \sqrt{2} + 2\sqrt{2}-\sqrt{2} = 6$$

$$\rightarrow A^2 = 6 \rightarrow A = \sqrt{6} \quad (II)$$

$$\frac{(II)}{A=\sqrt{6}} \rightarrow f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = 2A + 4 = 2\sqrt{6} + 4 \rightarrow f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = 4 + 2\sqrt{6}$$

$$\left\{ \begin{array}{l} 2f(x) - 3f(-x) = 4x^2 - x \xrightarrow{x^2} 4f(x) - 9f(-x) = 16x^2 - 4x \\ 2f(-x) - 3f(x) = 4x^2 + x \xrightarrow{x^2} 4f(-x) - 9f(x) = 16x^2 + 4x \end{array} \right.$$

$$\xrightarrow{\text{ضرب در ۳}} -9f(x) = 12x^2 + 4x \rightarrow f(x) = -\frac{4}{9}x^2 - \frac{4}{9}x$$

$$\rightarrow f(x) = -\frac{4}{9}x^2 - \frac{4}{9}x = -\frac{4}{9}x^2 - \frac{4}{9}x$$

$$\left. \begin{array}{l} x=0 \rightarrow 2f(0) - 0 = 4m - 1 \rightarrow 2f(0) = 4m - 1 \rightarrow f(0) = 2m - \frac{1}{2} \\ x=1 \rightarrow (0 \times f(1)) + 9f(0) = 16 + 4m + 4m - 1 \rightarrow 9f(0) = 16 + 8m - 1 \end{array} \right\} \rightarrow 9(2m - \frac{1}{2}) = 15 + 8m$$

$$\rightarrow 18m - \frac{9}{2} = 15 + 8m \rightarrow 10m = \frac{39}{2} \rightarrow m = \frac{39}{20}$$

$$m = \frac{39}{20} \rightarrow 2f(0) = \frac{39}{10} - 1 \rightarrow 2f(0) = \frac{29}{10} \rightarrow f(0) = \frac{29}{20}$$

$$\rightarrow f(0) = \frac{29}{20}$$

$$\left. \begin{array}{l} f(x) = ax + b \\ f(\frac{1}{x}) = a(\frac{1}{x}) + b = \frac{a}{x} + b \end{array} \right\} \rightarrow f(x) + f(\frac{1}{x}) = ax + b + \frac{a}{x} + b = ax + \frac{a}{x} + 2b$$

$$\frac{3x^2 - 12x + 12}{x^2} \xrightarrow{\text{ضرب در } x^2} 3x^2 - 12x + 12 = ax^2 + \frac{a}{x} + 2b$$

$$\left\{ \begin{array}{l} a=3 \rightarrow a=3 \\ b=-12 \rightarrow b=-12 \end{array} \right. \rightarrow f(x) = 3x - 12 \rightarrow f(-1) = -3 - 12 = -15$$

$$\rightarrow f(-1) = -15$$

$$\left. \begin{aligned} f(x) &= (x-1)\sqrt{1-x} \rightarrow 1-x \geq 0 \rightarrow x \leq 1 \rightarrow D_f = (-\infty, 1] \\ g(x) &= -\sqrt{(1-x)^3} \rightarrow (1-x)^3 \geq 0 \rightarrow 1-x \geq 0 \rightarrow x \leq 1 \rightarrow D_g = (-\infty, 1] \end{aligned} \right\} \rightarrow D_f = D_g$$

این دامنه ها برابرند باید مطمئن شویم آیا این دو تابع برابری می کنند.

$$g(x) = -\sqrt{(1-x)^3} \xrightarrow{1-x = t} g(x) = -\sqrt{t^3} = -(1-x)\sqrt{1-x} = (x-1)\sqrt{1-x}$$

$$\rightarrow g(x) = f(x) \rightarrow g(x) = f(x) \text{ (دو تابع برابرند)}$$

$$\left. \begin{aligned} f(x) &= x\sqrt{x} \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty) \\ g(x) &= \sqrt{x^3} \rightarrow x^3 \geq 0 \rightarrow x \geq 0 \rightarrow D_g = [0, +\infty) \end{aligned} \right\} \rightarrow D_f = D_g$$

بسیار ساده هر دو تابع برابرند و دامنه های آنها یکسان است پس این دو تابع باید یکدیگر برابرند.

$$g(x) = \sqrt{x^3} = \sqrt{x^2 \times x} = \sqrt{x^2} \times \sqrt{x} \xrightarrow{x \geq 0} g(x) = x\sqrt{x}$$

$$g(x) = f(x)$$

(دو تابع برابرند)

$$\left. \begin{aligned} f(x) &= [\sqrt{x} - \sqrt{x}] \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty) \\ g(x) &= \sqrt{x-x} \rightarrow x-x \geq 0 \xrightarrow{x \geq 0} g(x) = 0 \end{aligned} \right\} \rightarrow D_g = [0, +\infty)$$

در اینجا باید دقت کرد که هر دو تابع در تمام دامنه های خود صفر هستند.

$$\rightarrow D_g = D_f \rightarrow \text{این دو تابع در تمام دامنه های خود برابرند.}$$

$$\rightarrow f(x) = g(x) \text{ (دو تابع برابرند)}$$

برای $x < 0$ $g(x) = \sqrt{x-x} = \sqrt{-x}$ که در این صورت $x-x < 0$ و ریشه یاقبض نمی شود.

$$\left. \begin{aligned} x = \frac{1}{4} \rightarrow f(x) = [\frac{1}{2} - \frac{1}{2}] = 0 \rightarrow f(x) = g(x) \\ x = 1 \rightarrow f(x) = [1 - 1] = 0 \rightarrow f(x) = g(x) \end{aligned} \right\} \rightarrow f(x) = g(x)$$

$$a) \left(\frac{f+g}{f-g} \right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} \Rightarrow \begin{cases} f(1)=2 \\ g(1)=1 \end{cases} \rightarrow \frac{2+1}{2-1} = \frac{3}{1} = 3$$

$$b) \left(\frac{f+g}{f-g} \right)(2) = \frac{f(2)+g(2)}{f(2)-g(2)} \xrightarrow{\begin{cases} f(2)=3 \\ g(2)=3 \end{cases}} \left(\frac{f+g}{f-g} \right)(2) = \frac{3+3}{3-3} = \frac{6}{0} \text{ تعریف نشده است}$$

$$c) f(0) \cdot g(-1) = 0 \times 1 = 0$$

$$a) g(x) = f\left(\frac{1}{x}-1\right) \rightarrow \begin{cases} \frac{1}{x}-1 \geq 1 \rightarrow x \leq \frac{1}{2} \\ \frac{1}{x}-1 = -1 \rightarrow x \geq 0 \\ \frac{1}{x}-1 = 0 \rightarrow x = \frac{1}{2} \\ \frac{1}{x}-1 = -2 \rightarrow x = \frac{1}{3} \\ \frac{1}{x}-1 = 3 \rightarrow x = \frac{1}{4} \end{cases} \rightarrow g(x) = \left\{ \left(\frac{1}{2}, 1\right), \left(\frac{1}{3}, 0\right), \left(\frac{1}{4}, -1\right), \left(\frac{1}{5}, -2\right), \left(\frac{1}{6}, -3\right) \right\}$$

$$b) h(x) = f(|x|) \rightarrow |x| \geq 0 \rightarrow \begin{cases} x=0 \rightarrow h(x) = f(0) = 0 \\ x=2 \rightarrow x=\pm 2 \rightarrow h(x) = h(-2) = f(2) = 1 \\ x=3 \rightarrow x=\pm 3 \rightarrow h(x) = h(-3) = f(3) = -1 \end{cases}$$

$$\rightarrow h(x) = \{(-3, -1), (-2, 1), (0, 0), (2, 1), (3, -1)\}$$