

$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ 3a+2x & ; x \leq 1 \end{cases} \quad 2f(x) = f(-2) + a \quad a = ?$$

$$f(x) = (a+1)x \Rightarrow xa + x \rightarrow 2f(x) = 14a + 14$$

$$f(-2) = 3a - 2 \quad \Rightarrow \quad 14a + 14 = 3a - 2 + a \Rightarrow 10a = -16 \Rightarrow a = \frac{-16}{10} = -\frac{8}{5}$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} \quad \text{و} \quad f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = ?$$

$$f(\sqrt{2}-\sqrt{3}) = \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 \quad \xrightarrow{A} \quad (\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}) + \left(\frac{1}{\sqrt{2-\sqrt{3}}} + \frac{1}{\sqrt{2+\sqrt{3}}}\right) + 2$$

$$f(\sqrt{2}+\sqrt{3}) = \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 \quad \xrightarrow{B} \quad \frac{\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}}{\sqrt{(2-\sqrt{3})(2+\sqrt{3})}} \rightarrow B = \frac{\sqrt{4}}{1} = \sqrt{4}$$

$$\rightarrow A^2 = 4 + 2\sqrt{4-3} = 4 + 2 \times 1 = 6 \quad \rightarrow A = \sqrt{6} \quad \rightarrow B = \sqrt{4} = 2$$

$$A+B+2 = \sqrt{6} + 2 + 2 = \sqrt{6} + 4$$

$$2f(x) - 3f(-x) = 4x^2 - x \quad \text{و} \quad f(x) = ?$$

$$\begin{cases} (2f(x) - 3f(-x) = 4x^2 - x) \times 2 \rightarrow \begin{cases} 4f(x) - 6f(-x) = 8x^2 - 2x \\ -6f(x) + 4f(-x) = 12x^2 + 3x \end{cases} \rightarrow -2f(x) = 10x^2 + x \\ (-3f(x) + 2f(-x) = 4x^2 + x) \times 3 \rightarrow \begin{cases} -9f(x) + 6f(-x) = 12x^2 + 3x \end{cases} \rightarrow -3f(x) = 10x^2 + x \end{cases}$$

$$\rightarrow f(x) = \frac{10x^2 + x}{-3} \rightarrow f(x) = -\frac{10}{3}x^2 - \frac{1}{3}x$$

$$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1$$

$$x=0 \rightarrow (2f(0) - 3f(2)) = 4(0)^2 - m(0) + 3m - 1 \rightarrow 2f(0) - 3f(2) = 3m - 1$$

$$x=-2 \rightarrow 2f(-2) - 3f(0) = 4(-2)^2 - m(-2) + 3m - 1 \rightarrow 2f(-2) - 3f(0) = 15m - 5$$

$$\rightarrow 2f(0) = 9m - 1 \quad \rightarrow 2f(-2) = 15m - 5 \quad \rightarrow 9m - 1 = 15m - 5 \quad \rightarrow 6m = 4 \quad \rightarrow m = \frac{2}{3}$$

$$\rightarrow 2f(0) = 9 \times \frac{2}{3} - 1 \rightarrow \frac{12}{2} - \frac{1}{2} = \frac{11}{2} \rightarrow f(0) = \frac{11}{4}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 11x + 3}{x} \quad \text{و} \quad f(-1) = ?$$

$$f\left(\frac{1}{x}\right) = a\left(\frac{1}{x}\right) + b = \frac{a}{x} + b \rightarrow f(x) + f\left(\frac{1}{x}\right) = (ax + b) + \left(\frac{a}{x} + b\right) \rightarrow ax + \frac{a}{x} + 2b = \frac{3x^2 - 11x + 3}{x}$$

$$\rightarrow \frac{3x^2 - 11x + 3}{x} \rightarrow a = 3 \quad \text{و} \quad 2b = -11 \rightarrow b = -\frac{11}{2} \rightarrow f(x) = 3x - \frac{11}{2} \rightarrow f(-1) = -\frac{17}{2}$$

$$\begin{cases} f(x) = (x-1)\sqrt{1-x} \\ g(x) = -\sqrt{(1-x)^3} \end{cases} \xrightarrow{\text{بررسی دامنه‌ها}} \begin{aligned} 1-x > 0 &\rightarrow x < 1 \rightarrow D_f = (-\infty, 1] \\ (1-x)^3 > 0 &\rightarrow 1-x > 0 \rightarrow D_g = (-\infty, 1] \end{aligned} \Rightarrow \text{دامنه‌ها برابر است}$$

$$\left. \begin{aligned} x = -3 &\rightarrow f(x) = (-4)(2) = -8 \\ &\rightarrow g(x) = -\sqrt{(-4)^3} = -8 \\ x = -1 &\rightarrow f(x) = (-2)(2) = -4 \\ &\rightarrow g(x) = -\sqrt{(-2)^3} = -4 \end{aligned} \right\} \begin{aligned} &\text{دو تابع مساوی است} \\ &f(x) = g(x) \end{aligned}$$

$$\begin{cases} f(x) = x\sqrt{x} \\ g(x) = \sqrt{x^3} \end{cases} \xrightarrow{\text{بررسی دامنه‌ها}} \begin{aligned} x &\geq 0 \rightarrow D_f = [0, +\infty) \\ x^3 &\geq 0 \rightarrow x \geq 0 \rightarrow D_g = [0, +\infty) \end{aligned} \Rightarrow \text{دامنه‌ها برابر است}$$

$$\left. \begin{aligned} x = 2 &\rightarrow f(x) = 2\sqrt{2} \\ &\rightarrow g(x) = \sqrt{2^3} = 2\sqrt{2} \\ x = 4 &\rightarrow f(x) = 8 \\ &\rightarrow g(x) = \sqrt{4^3} = 8 \end{aligned} \right\} \begin{aligned} &\text{دو تابع مساوی است} \\ &f(x) = g(x) \end{aligned}$$

$$\begin{cases} f(x) = [\sqrt{x} - \sqrt{x}] \\ g(x) = \sqrt{x - |x|} \end{cases} \xrightarrow{\text{بررسی دامنه‌ها}} \begin{aligned} x &\geq 0 \rightarrow D_f = [0, +\infty) \\ x - |x| &\geq 0 \rightarrow x \geq |x| \rightarrow D_g = [0, +\infty) \end{aligned} \Rightarrow \text{دامنه‌ها برابر است}$$

$$\left. \begin{aligned} x = 4 &\rightarrow f(x) = 2 - 2 = 0 \\ &\rightarrow g(x) = \sqrt{4 - 4} = 0 \\ x = 9 &\rightarrow f(x) = 3 - 3 = 0 \\ &\rightarrow g(x) = \sqrt{9 - 9} = 0 \end{aligned} \right\} \begin{aligned} &\text{دو تابع مساوی است} \\ &f(x) = g(x) \end{aligned}$$

$$D_f = \{(1, 2), (2, 3), (3, 1), (0, 0)\} \quad D_g = \{(-1, 1), (1, 1), (2, 3), (3, 0), (0, 2), (\sqrt{2}, 1)\}$$

الف) $\left(\frac{f+g}{f-g}\right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} \rightarrow \frac{2+1}{2-1} = 3 \rightarrow \text{جواب}$

ب) $\left(\frac{f+g}{f-g}\right)(2) = \frac{f(2)+g(2)}{f(2)-g(2)} \rightarrow \frac{3+3}{3-3} = \frac{6}{0} \rightarrow \text{تعریف نشده} \rightarrow \text{جواب}$

ج) $f(0) \cdot g(-1) = f(0) = 0 \cdot g(-1) = 1 \rightarrow f(0) \cdot g(-1) = 0 \rightarrow \text{جواب}$

$$D_h = \{(2, 1), (-1, 4), (0, 0), (-2, 2), (3, -1)\}$$

الف) $g(x) = f(x-1) \rightarrow \begin{aligned} 3x-1 &= 2 \rightarrow x = 1 \\ 3x-1 &= -1 \rightarrow x = 0 \\ 3x-1 &= 0 \rightarrow x = \frac{1}{3} \\ 3x-1 &= -2 \rightarrow x = -\frac{1}{3} \\ 3x-1 &= 3 \rightarrow x = \frac{4}{3} \end{aligned}$

ب) $h(x) = f(|x|) \rightarrow \begin{aligned} |x| &= 2 \rightarrow x = \pm 2 \\ |x| &= -1 \rightarrow \text{غیرممکن} \\ |x| &= 0 \rightarrow x = 0 \\ |x| &= 3 \rightarrow x = \pm 3 \end{aligned}$

ج) $h(x) = (2, 1), (-2, 1), (0, 0), (3, -1), (-3, -1) \rightarrow \text{جواب}$