

$$f(x) = \begin{cases} (a+1)(x+1) & ; x > 1 \\ 3a+2x & ; x \leq 1 \end{cases} \quad 2f(x) = f(-1) + a \quad a = ?$$

$$f(x) = (a+1)x \Rightarrow xa + x \rightarrow 2f(x) = 14a + 14$$

$$f(-1) = 3a - 4 \quad \Rightarrow \quad 14a + 14 = 3a - 4 + a \Rightarrow 10a = -18$$

$$\rightarrow a = \frac{-18}{10} = \frac{-9}{5} \rightarrow \text{جواب}$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} \quad \text{و} \quad f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = ?$$

$$f(\sqrt{2}-\sqrt{3}) = \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2$$

$$f(\sqrt{2}+\sqrt{3}) = \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2$$

$$\rightarrow A = \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} \quad B = \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} \quad C = 4$$

$$\rightarrow A+B+C = 2\sqrt{6} + 4 \rightarrow \text{جواب}$$

$$2f(x) - 3f(-x) = 4x^2 - x \quad \text{و} \quad f(x) = ?$$

$$\begin{cases} (2f(x) - 3f(-x) = 4x^2 - x) \times 2 \rightarrow 4f(x) - 6f(-x) = 8x^2 - 2x \\ (-3f(x) + 2f(-x) = 4x^2 + x) \times 3 \rightarrow -9f(x) + 6f(-x) = 12x^2 + 3x \end{cases}$$

$$\rightarrow -5f(x) = 10x^2 + x \rightarrow f(x) = -2x^2 - \frac{1}{5}x \rightarrow \text{جواب}$$

$$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1$$

$$x=0 \rightarrow (2f(0) = 3m-1) \times 2 \rightarrow 4f(0) = 6m-2$$

$$x=-2 \rightarrow 4f(0) = 6m+1 \rightarrow 4f(0) = 6m+1$$

$$6m-2 = 6m+1 \rightarrow 9m = 3 \rightarrow m = \frac{1}{3}$$

$$\rightarrow 4f(0) = 4 \times \frac{1}{3} - 2 \rightarrow \frac{4}{3} - \frac{6}{3} = -\frac{2}{3} \rightarrow f(0) = -\frac{1}{3} \rightarrow \text{جواب}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 11x + 3}{x} \quad \text{و} \quad f(-1) = ?$$

$$f\left(\frac{1}{x}\right) = a\left(\frac{1}{x}\right) + b = \frac{a}{x} + b \rightarrow f(x) + f\left(\frac{1}{x}\right) = (ax+b) + \left(\frac{a}{x}+b\right) \rightarrow ax + \frac{a}{x} + 2b = \frac{3x^2 - 11x + 3}{x}$$

$$\rightarrow a = 3 \quad 2b = -11 \rightarrow b = -\frac{11}{2} \rightarrow f(x) = 3x - \frac{11}{2} \rightarrow f(-1) = -9 \rightarrow \text{جواب}$$

$$\begin{cases} f(x) = (x-1)\sqrt{1-x} \\ g(x) = -\sqrt{(1-x)^3} \end{cases} \xrightarrow{\text{بررسی دامنه‌ها}} \begin{aligned} 1-x > 0 &\rightarrow x < 1 \rightarrow D_f = (-\infty, 1] \\ (1-x)^3 > 0 &\rightarrow 1-x > 0 \rightarrow D_g = (-\infty, 1] \end{aligned} \Rightarrow \text{دامنه‌ها برابر است}$$

$$\left. \begin{aligned} x = -3 &\rightarrow f(x) = (-4)(2) = -8 \checkmark \\ &\rightarrow g(x) = -\sqrt{(-4)^3} = -8 \checkmark \\ x = -1 &\rightarrow f(x) = (-2)(2) = -4 \checkmark \\ &\rightarrow g(x) = -\sqrt{(-2)^3} = -2\sqrt{2} \checkmark \end{aligned} \right\} \begin{aligned} &\text{دو تابع مساوی است} \\ &f(x) = g(x) \end{aligned}$$

$$\begin{cases} f(x) = x\sqrt{x} \\ g(x) = \sqrt{x^3} \end{cases} \xrightarrow{\text{بررسی دامنه‌ها}} \begin{aligned} x &\geq 0 \rightarrow D_f = [0, +\infty) \\ x^3 &\geq 0 \rightarrow x \geq 0 \rightarrow D_g = [0, +\infty) \end{aligned} \Rightarrow \text{دامنه‌ها برابر است}$$

$$\left. \begin{aligned} x = 2 &\rightarrow f(x) = 2\sqrt{2} \checkmark \\ &\rightarrow g(x) = \sqrt{2^3} = \sqrt{8} = 2\sqrt{2} \checkmark \\ x = 4 &\rightarrow f(x) = 4\sqrt{4} = 8 \checkmark \\ &\rightarrow g(x) = \sqrt{4^3} = \sqrt{64} = 8 \checkmark \end{aligned} \right\} \begin{aligned} &\text{دو تابع مساوی است} \\ &f(x) = g(x) \end{aligned}$$

$$\begin{cases} f(x) = [\sqrt{x} - \sqrt{x}] \\ g(x) = \sqrt{x - |x|} \end{cases} \xrightarrow{\text{بررسی دامنه‌ها}} \begin{aligned} x &\geq 0 \rightarrow D_f = [0, +\infty) \\ x - |x| &> 0 \rightarrow x > |x| \rightarrow D_g = [0, +\infty) \end{aligned} \Rightarrow \text{دامنه‌ها برابر است}$$

$$\left. \begin{aligned} x = 4 &\rightarrow f(x) = 2 - 2 = 0 \checkmark \\ &\rightarrow g(x) = \sqrt{4 - 4} = 0 \checkmark \\ x = 9 &\rightarrow f(x) = 3 - 3 = 0 \checkmark \\ &\rightarrow g(x) = \sqrt{9 - 9} = 0 \checkmark \end{aligned} \right\} \begin{aligned} &\text{دو تابع مساوی است} \\ &f(x) = g(x) \end{aligned}$$

$$D_f = \{(1, 2), (2, 3), (3, 1), (0, 0)\} \quad D_g = \{(-1, 1), (1, 1), (2, 3), (3, 0), (0, 2), (\sqrt{2}, 1)\}$$

الف)  $\left(\frac{f+g}{f-g}\right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} \rightarrow \frac{2+1}{2-1} = 3 \rightarrow \text{جواب}$

ب)  $\left(\frac{f+g}{f-g}\right)(2) = \frac{f(2)+g(2)}{f(2)-g(2)} \rightarrow \frac{3+3}{3-3} = \frac{6}{0} \rightarrow \text{صورت نشده} \rightarrow \text{جواب}$

ج)  $f(0) \cdot g(-1) = f(0) = 0 \cdot g(-1) = 1 \rightarrow f(0) \cdot g(-1) = 0 \rightarrow \text{جواب}$

$$D_h = \{(2, 1), (-1, 4), (0, 0), (-2, 2), (3, -1)\}$$

الف)  $g(x) = f(x-1) \rightarrow 3x-1=2 \rightarrow x=1$  و  $3x-1=-1 \rightarrow x=0$  و  $3x-1=0 \rightarrow x=\frac{1}{3}$  و  $3x-1=-2 \rightarrow x=-\frac{1}{3}$  و  $3x-1=3 \rightarrow x=\frac{4}{3} \rightarrow g(x) = \left\{ (1, 0), (0, 4), \left(\frac{1}{3}, 0\right), \left(-\frac{1}{3}, 0\right), \left(\frac{4}{3}, -1\right) \right\}$

ب)  $h(x) = f(|x|) \rightarrow |x|=2 \rightarrow x=\pm 2$  و  $|x|=-1 \rightarrow \text{غیرممکن}$  و  $|x|=0 \rightarrow x=0$  و  $|x|=1 \rightarrow x=\pm 1$  و  $|x|=3 \rightarrow x=\pm 3 \rightarrow h(x) = \{(2, 1), (-2, 1), (0, 0), (3, -1), (-3, -1)\} \rightarrow \text{جواب}$