

۲. آزمون عالی نوشتار

$$f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 3a + 2x & x < 1 \end{cases}$$

$$f(1) = f(-2) + a \Rightarrow 1(a+1) + 1 = 3a - 2 + a \Rightarrow 10a = -1 \Rightarrow a = -\frac{1}{10}$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} = A \Rightarrow A^2 = 2 + \sqrt{3} + 2 - \sqrt{3} + 2\sqrt{(2+\sqrt{3})(2-\sqrt{3})} = 4 \Rightarrow A = \sqrt{4}$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = 2(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}) + 2 = 2\sqrt{4} + 2 = 4 + 2 = 6$$

$$\frac{1}{\sqrt{2-\sqrt{3}}} = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{(2-\sqrt{3})(2+\sqrt{3})}} = \sqrt{2+\sqrt{3}}$$

$$\frac{1}{\sqrt{2+\sqrt{3}}} = \sqrt{2-\sqrt{3}}$$

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$$f(x) - 3f(-x) = 4x^2 - 2x \Rightarrow f(x) - 4f(x) = 12x^2 - 2x$$

$$f(-x) - 3f(x) = 4x^2 + 2x \Rightarrow -4f(x) + 4f(-x) = 12x^2 + 2x$$

$$-4f(x) = 10x^2 + 2x \Rightarrow f(x) = -\frac{5}{2}x^2 - \frac{1}{2}x$$

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$$(x+2)f(x) - 3xf(x+2) = 2x^2 - mx + 3m - 1$$

$$x=0 \Rightarrow 2f(0) = 3m - 1 \Rightarrow 10f(0) = 15m - 5$$

$$x=-2 \Rightarrow 4f(0) = 15 + 6m \Rightarrow \frac{10f(0) = 15 + 6m}{10f(0) = 5}$$

$$f(0) = \frac{5}{10} = \frac{1}{2}$$

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$$f(x) + f\left(\frac{1}{x}\right) = \frac{x^2 - 12x + 3}{x}$$

$$x=-1 \Rightarrow 2f(-1) = \frac{1}{-1} \Rightarrow f(-1) = -\frac{1}{2}$$

$$ax + b + \frac{a}{x} + b = \frac{x^2 - 12x + 3}{x} \Rightarrow \frac{ax^2 + bx + a}{x} = \frac{x^2 - 12x + 3}{x} \Rightarrow a=1, b=-4$$

$$f(x) = x - 4 \Rightarrow f(-1) = -5$$

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$\begin{cases} f(n) = (n-1)\sqrt{1-n} \Rightarrow 1, n \checkmark \\ g(n) = -\sqrt{(1-n)^3} \Rightarrow (1-n)^3 \geq 0 \Rightarrow 1, n \checkmark \end{cases}$ $\begin{aligned} f(-2) &= -1 \checkmark \\ g(-2) &= -1 \checkmark \\ f(0) &= -1 \checkmark \\ g(0) &= -1 \checkmark \end{aligned}$ در تابع مساوی هستند	۶
$\begin{cases} f(n) = n\sqrt{n} \\ g(n) = \sqrt{n^3} \end{cases}$ $\begin{aligned} n \geq 0 \checkmark & \quad f(0) = 0 \checkmark \quad f(4) = 8 \checkmark \\ n \geq 0 \Rightarrow n \geq 0 \checkmark & \quad g(0) = 0 \checkmark \quad g(4) = 8 \checkmark \end{aligned}$ در تابع مساوی هستند	۷
$\begin{cases} f(n) = [\sqrt{n} - \lfloor \sqrt{n} \rfloor] \\ g(n) = \sqrt{n - n } \end{cases}$ $\begin{aligned} n \geq 0 \checkmark & \quad f(1) = 0 \checkmark \quad f(4) = 0 \checkmark \\ n \geq n \Rightarrow n \geq 0 \checkmark & \quad g(1) = 0 \checkmark \quad g(4) = 0 \checkmark \end{aligned}$ در تابع مساوی هستند	۸
الف) $\left(\frac{f+g}{f-g}\right)(1) = \frac{2+1}{2-1} = 3$ ب) $\left(\frac{f+g}{f-g}\right)(2) = \frac{3+3}{3-3}$ تعریف نشده ج) $f(0)g(-1) = 0 \times -1 = 0$	۹
الف) $g(n) = f(3n-1)$ $g(n) = \left\{ (1, 1), (0, 4), \left(\frac{1}{3}, 5\right), \left(-\frac{1}{3}, 2\right), \left(\frac{2}{3}, -1\right) \right\}$ ب) $h(n) = f(n)$ $h(n) = \left\{ (-2, 1), (2, 1), (0, 2), (4, -1), (-4, 1) \right\}$	۱۰