

$$f(a)(v) = 2a - 9 + a$$

$$f(a+1) - f(a-1)$$

$$\frac{2}{3}a + \frac{2}{3} = a - 1$$

$$\frac{2}{3}a + \frac{2}{3} = 0 \Rightarrow a = -1$$

۱

$$f(2-\sqrt{3}) + f(2+\sqrt{3})$$

$$\sqrt{2-\sqrt{3}}$$

$$\sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 + \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 = 2 \left(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + 2 \right) = 2\sqrt{4} + 4$$

$$(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}})^2 = 2 - \sqrt{3} + 2 + \sqrt{3} + 2 = 4 \Rightarrow \sqrt{4} = 2$$

۲

$$x \begin{cases} 4f(x) - 3f(-x) = 4x^2 - 2 \\ 4f(-x) - 3f(x) = 4x^2 + 2 \end{cases}$$

$$x \begin{cases} 4f(x) - 3f(-x) = 4x^2 - 2 \\ 4f(-x) - 3f(x) = 4x^2 + 2 \end{cases}$$

$$4f(x) - 3f(-x) = 4x^2 - 2$$

$$4f(-x) - 3f(x) = 4x^2 + 2$$

$$-2f(x) = 4x^2 + 2 \Rightarrow f(x) = -x^2 - \frac{1}{2}$$

۳

$$x = -2 \Rightarrow 4f(-2) = 14 + 2m - 1 \Rightarrow f(-2) = \frac{13}{4} + \frac{m}{2}$$

$$x = 0 \Rightarrow 4f(0) = 2m - 1 \Rightarrow f(0) = \frac{2m-1}{4}$$

$$\frac{13}{4} + \frac{m}{2} = \frac{2m-1}{4}$$

$$13 + 2m = 2m - 1$$

$$14 = 0$$

$$\frac{13}{4} + \frac{m}{2} = \frac{2m-1}{4}$$

۴

$$f(x) + f\left(\frac{1}{x}\right) = \frac{2x^2 - 12x + 7}{x}$$

$$ax^2 + b + \frac{a}{x} + \frac{b}{x} = \frac{ax^2 + bx + a + b}{x} = \frac{2x^2 - 12x + 7}{x}$$

$$\begin{cases} b = -4 \\ a = 2 \end{cases}$$

$$2x - 4 \xrightarrow{-1} -4$$

$$f(-1) = -4$$

۵

$$D_f \rightarrow 1-x > 0 \rightarrow x < 1$$

$$D_g \rightarrow (1-x) > 0 \rightarrow x < 1$$

$$f(x) = \frac{x}{(x-1)\sqrt{1-x}}$$

$$g(x) = -\sqrt{(1-x)^3} = -|1-x|\sqrt{1-x} \xrightarrow{n \leq 1} -(1-x)\sqrt{1-x}$$

$$g(x) = -\sqrt{(1-x)^3} \rightarrow -|1-x|\sqrt{1-x}$$

پس فو درج اولیه را می بینیم

$$= x-1\sqrt{1-x} = f(x) \checkmark$$

$$\begin{cases} f(x) = x\sqrt{x} & D_f = [0, +\infty) \\ g(x) = \sqrt{x^3} & D_g = [0, +\infty) \end{cases}$$

مابین آنها برابر است

$$f(x) = [\sqrt{x} - [\sqrt{x}]] \rightarrow x > 0 \quad D_f = [0, +\infty)$$

$$g(x) = \sqrt{x-|x|} \quad g(x) = 0$$

برابر است

$$\left(\frac{f+g}{f-g}\right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{1+0}{1-0} = 1$$

$$g(n) = f(n-1) = \left\{ \left(1, \frac{1}{2}\right), \left(0, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right) \right\}$$

$$h(n) = f(|n|) = \left\{ (1, 1), \left(-\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right), \left(-\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right) \right\} = \left\{ (1, 1), \left(-\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right), \left(-\frac{1}{2}, \frac{1}{2}\right) \right\}$$

$$f(n) = (n-1)\sqrt{1-n} \rightarrow Df = 1-n \geq 0 \rightarrow n \leq 1 \rightarrow Df = (-\infty, 1]$$

$$g(n) = -\sqrt{(1-n)^3} \rightarrow Dg = (1-n)^2 \geq 0 \rightarrow n \leq 1 \rightarrow Dg = (-\infty, 1]$$

(4)

(دانشگاه) شرط اول برقرار است! $Df \subseteq Dg$ ✓

$$g(n) = -\sqrt{(1-n)^3} = -\sqrt{(1-n)^2(1-n)} = -|1-n|\sqrt{1-n} \xrightarrow{n \leq 1} -(1-n)\sqrt{1-n} = (n-1)\sqrt{1-n}$$

$$f - g(n) \text{ ✓ (ضابطه ها) شرط دوم برقرار است!} \rightarrow \boxed{f(n) = g(n)}$$