

$$f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 3a+2x & x \leq 1 \end{cases} \quad a=1$$

$$2f(5) = f(2) + a$$

$$2((a+1)5) = 3a+2(2) + a$$

$$10a+10 = 3a-4+a = 4a-4$$

$$1 \cdot a = -18 \\ \Rightarrow a = -18$$

1

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad a = 2 - \sqrt{3} \quad b = 2 + \sqrt{3}$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = ? \quad ab = (2-\sqrt{3})(2+\sqrt{3}) = 1$$

$$(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab} \quad \sqrt{b} = \frac{1}{\sqrt{a}} \quad f(a) + f(b) = 2(\sqrt{a} + \sqrt{b}) + 2$$

$$= 2 - \sqrt{3} + 2 + \sqrt{3} + 2 = 6 \quad \sqrt{a} + \sqrt{b} = \sqrt{4}$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = 4 + 2\sqrt{4}$$

2

$$2(f(x) - 3f(-x)) = (4x^2 - x) = 4x^2 - 2x$$

$$-x \rightarrow \frac{2(f(x) - 3f(-x))}{-x} = \frac{4x^2 - x}{-x} = 12x^2 + 2x$$

$$-2f(x) = 12x^2 + 2x \Rightarrow f(x) = -6x^2 - x$$

$$2f(x) = -12x^2 - 2x$$

3

$$(x+2)f(x) = 3xf(x+2) = 4x^2 - mx + 3m-1$$

$$f(0) = 1 \quad x=0 \rightarrow 2f(0) = 3m-1$$

$$x=-2 \rightarrow 4f(0) = 14+2m+3m-1$$

$$4f(0) = 8m+14$$

$$f(0) = \frac{3m-1}{2}$$

$$\frac{3(\frac{8}{2}) - 1}{2} = \frac{10}{2} - \frac{1}{2} = \frac{9}{2}$$

$$2(3m-1) = 8m+14$$

$$6m-2 = 8m+14$$

$$2m = -16$$

$$\Rightarrow m = \frac{-16}{2} = -8$$

4

$$f \rightarrow \text{تابع خطی} \rightarrow f(x) = ax+b$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{ax^2 - 12x + 1}{x}$$

$$f(-1) = ?$$

$$= \frac{ax^2 - 12x + 1}{x} = \frac{a(-1)^2 - 12(-1) + 1}{-1}$$

$$= -a + 12 - 1 = -a + 11$$

$$f(x) = ax+b \rightarrow 3x-9 \quad f(-1) = \frac{3(-1)-9}{-1} = 6$$

$$f\left(\frac{1}{x}\right) = \frac{a}{x} + b$$

$$f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b$$

$$= ax + \frac{a}{x} + 2b$$

$$\Rightarrow a=12 \quad b=-9$$

5

$$-(1-x)^{\frac{1}{2}} \cdot x(1-x)^{\frac{1}{2}} = -x(1-x)^{\frac{1}{2}}$$

$$f(x) = (x-1)\sqrt{1-x} \quad x \leq 1 \quad 0 \leq x \leq 1$$

$$g(x) = -\sqrt{1-x}^3 \quad (1-x)^{\frac{3}{2}} \quad x \leq 0$$

$$-(1-x)^{\frac{3}{2}} \quad 1-x \geq 0 \quad x \leq 1$$

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$$f(x) = x\sqrt{x} \quad x \geq 0$$

$$g(x) = \sqrt{x^3} \quad x \geq 0$$

$$x^{\frac{3}{2}} \quad x \geq 0$$

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$$f(x) = [\sqrt{x} - [\sqrt{x}]] \quad x \geq 0$$

$$g(x) = \sqrt{x - [x]} \quad x - [x] \geq 0 \quad x \geq [x] \quad x \geq 0$$

$$x = [x]$$

$$1 \rightarrow [\sqrt{1} - [\sqrt{1}]]$$

$$[\sqrt{2,5} - [\sqrt{2,5}]]$$

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$$f = \{(1,2), (2,3), (3,4), (4,5), (5,6)\} \quad g = \{(1,1), (2,2), (3,3), (4,4), (5,5)\}$$

$$\frac{f+g}{f-g}(1) = \frac{3}{1} = 3$$

$$\frac{f+g}{f-g}(2) \rightarrow 5$$

$$f(5) \cdot g(-1) = 0 \cdot 1 = 0$$

$$f(x) = \{(2,1), (-1,4), (0,5), (-2,2), (3,-1)\}$$

$$g(x) = f(3x-1) = \{(1,1), (0,4), (\frac{1}{3},5), (-\frac{1}{3},2), (0,\frac{4}{3}), (-1,-1)\}$$

$$h(x) = f(|x|) = \{(2,1), (-2,1), (0,5), (-2,2), (0,2), (3,-1)\}$$

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