

$$2f(a) = f(-2) + a \Rightarrow 2(\sqrt{a} + \frac{1}{\sqrt{a}}) = 3a - 4 + a \Rightarrow 4a + 4 = 4a - 4$$

$$\Rightarrow 10a = -18 \Rightarrow a = -\frac{9}{5}$$

۱

$$\begin{cases} 2 - \sqrt{3} = a \\ 2 + \sqrt{3} = b \end{cases} \Rightarrow (2 - \sqrt{3})(2 + \sqrt{3}) = 1 \Rightarrow ab = 1 \Rightarrow b = \frac{1}{a}$$

$$\Rightarrow f(a) + f(b) = f(a) + f(\frac{1}{a}) = \sqrt{a} + \frac{1}{\sqrt{a}} + 2 + \sqrt{\frac{1}{a}} + \frac{1}{\sqrt{\frac{1}{a}}} + 2$$

۲

$$\Rightarrow f(a) + f(b) = 2(\sqrt{a} + \frac{1}{\sqrt{a}}) + 4 \quad ①$$

$$\sqrt{a} + \frac{1}{\sqrt{a}} = \sqrt{a} + \sqrt{\frac{1}{a}} = \sqrt{a} + \sqrt{b} = \sqrt{a+b+2\sqrt{ab}} = \sqrt{a+b+2} \quad ②$$

$$\sqrt{2\sqrt{a} + 2 + 2\sqrt{a} + 2} = \sqrt{6} \quad ③ \xrightarrow{① \text{ و } ②} f(a) + f(b) = 2\sqrt{6} + 4$$

۳

$$\begin{cases} 2f(x) - 3f(-x) = 4x^2 - x \\ 2f(-x) - 3f(x) = 4x^2 + x \end{cases} \Rightarrow -(f(x) + f(-x)) = 8x^2$$

$$\Rightarrow f(-x) = -(f(x) + 8x^2) \Rightarrow 2f(x) + 3f(x) + 2(4x^2 - x) = 4x^2 - x$$

$$\Rightarrow 5f(x) = -2 \cdot 4x^2 - x \Rightarrow f(x) = -\frac{8x^2}{5} - \frac{x}{5}$$

۴

$$\text{if } x=0 \Rightarrow 2f(0) = 3m - 1 \Rightarrow f(0) = \frac{3m-1}{2}$$

$$\text{if } x=-1 \Rightarrow 9f(0) = 16 + 2m - 1 \Rightarrow 9m - 3 = 2m + 16$$

$$\Rightarrow 7m = 19 \Rightarrow m = \frac{19}{7} \Rightarrow f(0) = \frac{2 \cdot \frac{19}{7} - 1}{2} = \frac{11}{7}$$

۵

$$f(x) = ax + b \Rightarrow f(x) + f(\frac{1}{x}) = ax + b + \frac{a}{x} + b =$$

$$ax + \frac{a}{x} + 2b = 3x + \frac{3}{x} - 12 \Rightarrow a=3, b=-6$$

$$\Rightarrow f(x) = 3x - 6 \Rightarrow f(-1) = -3 - 6 = -9$$

۶

$1-x \geq 0 \Rightarrow 1 \geq x \Rightarrow D_f = (-\infty, 1]$ $(1-x)^3 \geq 0 \Rightarrow 1-x \geq 0 \Rightarrow 1 \geq x \Rightarrow D_g = (-\infty, 1]$ $\left. \begin{array}{l} D_f = (-\infty, 1] \\ D_g = (-\infty, 1] \end{array} \right\} \Rightarrow D_f = D_g$ $g(x) = -\sqrt{(1-x)x(1-x)} = - 1-x \sqrt{1-x} \xrightarrow{1-x \geq 0} g(x) = f(x) = (x-1)\sqrt{1-x}$ دامنه‌ها برابر دارند و مقادیر هایشان یکسان عمل میکنند و پس برابر اند.	۶
$x \geq 0 \Rightarrow D_f = [0, +\infty)$ $x^3 \geq 0 \Rightarrow x \geq 0 \Rightarrow D_g = [0, +\infty)$ $\left. \begin{array}{l} D_f = [0, +\infty) \\ D_g = [0, +\infty) \end{array} \right\} \Rightarrow D_f = D_g$ $\sqrt{x^3} = \sqrt{x^2 \cdot x} = x \sqrt{x} \xrightarrow{x \geq 0} g(x) = f(x) = x\sqrt{x}$ دامنه‌ها برابر دارند و مقادیر هایشان یکسان عمل میکنند و پس برابر اند.	۷
$x \geq 0 \Rightarrow D_f = [0, +\infty)$ $x - x \geq 0 \Rightarrow x \geq x \Rightarrow x \geq 0 \Rightarrow D_g = [0, +\infty)$ $\left. \begin{array}{l} D_f = [0, +\infty) \\ D_g = [0, +\infty) \end{array} \right\} \Rightarrow D_f = D_g$ $if \ x \geq 0 \Rightarrow 0 \leq \sqrt{x} - [\sqrt{x}] < 1 \Rightarrow [\sqrt{x} - [\sqrt{x}]] = 0 \Rightarrow f(x) = 0$ $if \ x \geq 0 \Rightarrow x = x \Rightarrow \sqrt{x - x } = 0 \Rightarrow g(x) = 0$ دامنه‌ها برابر دارند و مقادیر هایشان یکسان عمل میکنند و پس برابر اند.	۸
الف) $f+g = \{(1,3)(2,6)(3,9)(0,0)\}$ $f-g = \{(1,1)(2,0)(3,-4)(0,-2)\}$ $\Rightarrow \left(\frac{f+g}{f-g} \right)(1) = \frac{3}{1} = 3$ ب) $\left(\frac{f+g}{f-g} \right)(2) = \frac{6}{0} \Rightarrow$ تعریف نمی‌شود ج) $f(0)g(-1) = 0 \times 1 = 0$	۹
الف) $g(x) = \{(1,1)(0,4)(\frac{1}{3},5)(-\frac{1}{3},2)(\frac{4}{3},-1)\}$ ب) $h(x) = \{(2,1)(-2,1)(0,5)(3,-1)(-3,-1)\}$	۱۰