

$$2f(\omega) = f(-2) + \alpha \quad ; \quad f(x) = \begin{cases} (\alpha+1)(x+2) & ; x > 1 \\ 3\alpha+2x & ; x < 1 \end{cases}$$

$$2(\alpha+1)(\omega+2) = 3\alpha+2(-2) + \alpha$$

$$(3\alpha+2)(7) = 3\alpha + \alpha - 4$$

$$17\alpha + 14 = 4\alpha - 4 \rightarrow 13\alpha = -18 \rightarrow \alpha = -\frac{18}{13} = -\frac{9}{\frac{13}{2}} \quad \text{جواب نهایی}$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 + \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = \frac{1}{\sqrt{2-\sqrt{3}}} = \sqrt{2+\sqrt{3}} \quad , \quad \frac{1}{\sqrt{2+\sqrt{3}}} = \sqrt{2-\sqrt{3}}$$

$$= 2\sqrt{2-\sqrt{3}} + 2\sqrt{2+\sqrt{3}} + 4 \quad , \quad \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} = \sqrt{4}$$

$$\Rightarrow 4 + 2\sqrt{4} \quad \text{جواب نهایی}$$

$$(2f(x) - 3f(-x)) = 4x^2 - 4x^2$$

$$(2f(-x) - 3f(x)) = 4x^2 + 4x^2$$

$$4f(x) - 4f(-x) = 8x^2 - 4x$$

$$4f(-x) - 4f(x) = 8x^2 + 4x$$

$$\rightarrow -2f(x) = 20x^2 + 4x \rightarrow f(x) = -2x^2 - \frac{x}{5} \quad \text{جواب نهایی}$$

$$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1$$

$$(0+2)f(0) - 3(0)f(2) = 3m - 1 \rightarrow 2f(0) = 3m - 1 \rightarrow f(0) = \frac{3m-1}{2}$$

$$x = -2 \Rightarrow 0 \cdot x f(-2) - 3(-2)f(0) = 4(-2)^2 - m(-2) + 3m - 1$$

$$4f(0) = 14 + 2m + 3m - 1 \rightarrow 4f(0) = 13 + 5m \rightarrow 4\left(\frac{3m-1}{2}\right) = 13 + 5m$$

$$2(3m-1) = 13 + 5m \rightarrow 6m - 2 = 13 + 5m \rightarrow m = 15$$

$$f(0) = \frac{3\left(\frac{15}{2}\right) - 1}{2} = \frac{45}{4} - \frac{1}{2} = \frac{43}{4} \quad \text{جواب نهایی}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x} \Rightarrow f\left(\frac{1}{x}\right) = \frac{a}{x} + b$$

$$f(x) = ax + b$$

$$f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b = ax + \frac{a}{x} + 2b$$

$$\frac{3x^2 - 12x + 3}{x} = 3x - 12 + \frac{3}{x} \rightarrow a = 3, 2b = -12 \Rightarrow b = -6 \rightarrow f(x) = 3x - 6$$

$$f(-1) = 3(-1) - 6 = -9 \quad \text{جواب نهایی}$$

$$f(x) = (x-1)\sqrt{1-x} \rightarrow x-1 = -(1-x) \rightarrow f(x) = -(1-x)\sqrt{1-x} \rightarrow f(x) = -(1-x)^{3/2}$$

$$g(x) = -\sqrt{(1-x)^3} \quad 1-x \geq 0 \rightarrow x \leq 1$$

$$\downarrow$$

$$(1-x)(\sqrt{1-x}) \rightarrow g(x) = -(1-x)\sqrt{1-x}$$

$$\underline{f(x) = g(x)} \rightarrow D_{\text{هر دو}} = (-\infty, 1]$$

$$f(x) = x\sqrt{x} \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty)$$

$$g(x) = \sqrt{x^3} \xrightarrow{x \geq 0} \sqrt{x^2 \cdot x} = \sqrt{x^2} \sqrt{x} = x\sqrt{x}$$

$$g(x) = x\sqrt{x} = f(x) \rightarrow \underline{f(x) = g(x)}$$

$$f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor]$$

$$g(x) = \sqrt{x - |x|} \rightarrow x - |x| \geq 0 \xrightarrow{x \geq 0} x - |x| = x - x = 0$$

(د تابع برابر هستند)

$$x \leq 0 \rightarrow x - |x| = x - (-x) = 2x < 0 \rightarrow D_g = [0, +\infty)$$

$$g(x) \xrightarrow{x \geq 0} = \sqrt{0} = 0$$

$$f(x) \neq g(x) \quad \text{مثلاً برای } x=2 \quad f(2) = \sqrt{2} - [\sqrt{2}] = \sqrt{2} - 1$$

$$f = \{(1,2), (2,3), (3,1), (0,0)\} \quad g = \{(-1,1), (1,1), (2,3), (3,5), (0,2), (\sqrt{2},1)\}$$

$$f(1) = 2 \quad g(1) = 1 \rightarrow (f+g)(1) = 2+1 = 3, \quad (f-g)(1) = f(1) - g(1) = 2-1 = 1$$

$$\text{جواب الف} \quad \frac{f+g}{f-g}(2) \rightarrow f(2) = 3, \quad g(2) = 3 \quad (f+g)(2) = 3+3 = 6, \quad (f-g)(2) = 3-3 = 0$$

$$\frac{f-g}{f+g} = 0$$

$$\text{جواب ج} \quad f(0)g(-1) \rightarrow f(0) = 0, \quad g(-1) = 1 \rightarrow 0 \times 1 = 0$$

$$f = \{(2,1), (-1,2), (0,5), (-2,2), (3,-1)\}$$

$$\text{الف} \quad g(x) = f(2x-1)$$

$$g = \{(1,1), (0,4), (\frac{1}{2}, 5), (-\frac{1}{2}, 2), (\frac{3}{2}, -1)\}$$

$$h(x) = f(|x|) \quad |x| \geq 0 \rightarrow |x| = 0 \rightarrow x = 0 \Rightarrow h(0) = f(0) = 5$$

$$|x| = 2 \rightarrow x = 2 \rightarrow h(2) = f(2) = 1$$

$$h = \{(-2,1), (0,5), (2,1), (-2,-1), (2,-1)\}$$

$$2x-1=2 \rightarrow 2x=3 \rightarrow x=\frac{3}{2} \rightarrow g(\frac{3}{2}) = f(2) = 1$$

$$2x-1=-1 \rightarrow 2x=0 \rightarrow x=0 \rightarrow g(0) = f(-1) = 2$$

$$2x-1=0 \rightarrow x=\frac{1}{2} \rightarrow g(\frac{1}{2}) = f(0) = 5$$

$$2x-1=5 \rightarrow 2x=6 \rightarrow x=3 \rightarrow g(3) = f(5) = -1$$

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