

سوال ① $f(x) = \sqrt{x+\varepsilon} \sqrt{x-\varepsilon} \xrightarrow[t=x+\varepsilon]{t=\sqrt{x-\varepsilon}} f(x) = \sqrt{t^2+\varepsilon+\varepsilon t} = \sqrt{(t+\varepsilon)^2}$

$g(x) = \sqrt{x-\varepsilon} \sqrt{x-\varepsilon} \xrightarrow[t=x+\varepsilon]{t=\sqrt{x-\varepsilon}} g(x) = \sqrt{t^2+\varepsilon-\varepsilon t} = \sqrt{(t-\varepsilon)^2} = |t-\varepsilon|$
 $\xrightarrow[t \geq \varepsilon]{t < \varepsilon} t-\varepsilon, \varepsilon-t$

$f(x)+g(x) = a+b \sqrt{x+c} \xrightarrow{C=-\varepsilon} C=-\varepsilon$
 $f(x)g(x) = x = 2\sqrt{x-\varepsilon} \xrightarrow{a+b=2} a+b=2$

$\frac{a+b}{-c} = \frac{2}{-\varepsilon} = -\frac{1}{\varepsilon}$

سوال ② $f(x) = \frac{x+2}{x^2-\varepsilon x+3} \quad g(x) = \{(2,1)(1,0)(3,\omega)(0,\varepsilon)\}$

$\frac{g}{f} = \left\{ \left(2, \frac{-\frac{1}{\varepsilon}}{\frac{3}{-1}}\right) \left(1, \frac{\frac{0}{1}}{\frac{3}{0}}\right) \left(3, \frac{\frac{\omega}{1}}{\frac{\omega}{0}}\right) \left(0, \frac{\frac{\varepsilon}{1}}{\frac{2}{3}}\right) \right\}$

$\Rightarrow \frac{g}{f} = \left\{ \left(2, -\frac{1}{\varepsilon}\right), (0, \varepsilon) \right\}$

سوال ③ $f(x) = \{(1,2)(3,-1)(\varepsilon,2)(-1,4)\} \quad g(x) = \{(-2,3)(1,-1)(3,2)(-1,0)\}$

الف $f(2x) = \left\{ \left(\frac{1}{2}, 2\right) \left(\frac{3}{2}, -1\right) \left(2, 2\right) \left(-\frac{1}{2}, 4\right) \right\}$ ب $f(x^2) = \left\{ (1,2)(\sqrt{3},-1)(2,2)(-2,2) \right\}$
 $(-1,2)(-\sqrt{3},-1)$

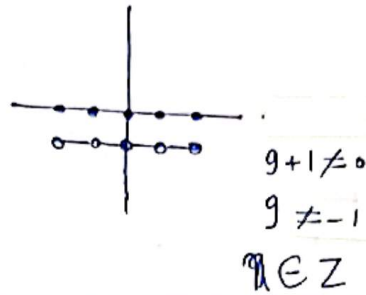
ج $2g^2(x)+1 = \left\{ (-2,9)(1,3)(3,9)(-1,1) \right\} \xrightarrow{\frac{2f}{g}} \left\{ (-1,-\varepsilon)(3,-1) \right\}$

سوال ④ $f(x) = \{(2,1)(3,1)(\varepsilon,9)(-2,0)\} \quad g(x) = \{(\omega,1)(2,3)(-2,4)(-1,2)(3,3)\}$

$h = 2f+g \quad h = \left\{ (-2,9)(2,\omega)(3,\omega) \right\}$

سوال ۵) $f(x) = \frac{\sqrt{3x} - x}{x-2}$ $g(x) = [x] + [-x]$ $\left(\frac{f}{g+1}\right)(x)$

$\sqrt{3x} - x \geq 0 \rightarrow 0 \leq x \leq 3 - \{2\}$



$D = \{0, 1, 3\}$

سوال ۶) $f(x) = \sqrt{5x - x^2 - 4}$ $g(x) = \{(1, 3), (2, 2), (4, 4), (5, 1)\}$

$h(x) = \frac{f(x)}{g(x)} = \left\{ \left(1, \frac{0}{3}\right), \left(2, \frac{\sqrt{2}}{2}\right), \left(4, \frac{0}{4}\right), \left(5, \frac{-4}{1}\right) \right\}$

$h(x) = \left\{ (1, 0), \left(2, \frac{\sqrt{2}}{2}\right), (4, 0) \right\}$

سوال ۷) $y = x^3 - 3x^2 + 3x$ $\Rightarrow x^3 - 3x^2 + 3x - y = 0$ این معادله برای هر $y \in \mathbb{R}$ طبق یک ریشه حقیقی x دارد در همین نیز ممکن است توانش نزدیک است

$R = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow yx^2 - y2x = 1 \Rightarrow yx^2 - 2yx - 1 = 0$

$x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \Rightarrow 4(y^2 + y) \geq 0 \rightarrow y(y+1) \geq 0$

$\Rightarrow R = (-\infty, -1] \cup [0, +\infty)$

الف) $y = x^2 - 4x + 10$

$x_{\min} = \frac{4}{2} = 2$ $y_{\min} = 6$

$R = [6, +\infty)$

ب) $y = -x^2 + 4x + 3$

$x_{\min} = \frac{4}{-2} = -2$ $y_{\min} = -5$

$R = (-\infty, 7]$

ج) $y = \sqrt{x^2 - 4x - 3}$

$x_{\min} = \frac{4}{2} = 2$ $y_{\min} = -5$

$R = [0, +\infty)$

$R = [0, +\infty)$

د) $y = \sqrt{4x - x^2}$

$x_{\min} = \frac{4}{2} = 2$ $y_{\min} = 2$

$R = [2, 2]$

$R = [0, 3]$

سوال ٩

$$الف) y = x^3 - 5x^2 + 2x + 1 \quad R = \mathbb{R}$$

$$ب) y = x^4 - 4x^2 + 3x + 2 \quad R = \mathbb{R}$$

$$ج) y = \sqrt{x^4 - 4x^2 + 5x + 1} \quad R = \sqrt{\mathbb{R}} \quad R = [0, +\infty)$$

$$د) (x^5 - 4x^2 + 3x + 1)^{\frac{1}{3}} \quad R = [0, +\infty)$$

$$الف) y = \frac{3x+1}{x-2} \quad x = -\frac{-2y-1}{y-3} = \frac{2y+1}{y-3} \Rightarrow y \neq 3 \quad R = \mathbb{R} - \{3\} \quad \text{سوال ١٥}$$

$$ب) y = \frac{5x+1}{x+1} \quad x = -\frac{y-1}{y-5} = \frac{1-y}{y-5} \Rightarrow y \neq 5 \quad R = \mathbb{R} - \{5\}$$