

۲. این بنیادی نرستی

$$\begin{aligned} t = \sqrt{x-4} \\ x = t^2 + 4 \end{aligned} \rightarrow \begin{cases} f(t) = \sqrt{t^2 + 4 + 4t} = \sqrt{(t+2)^2} = |t+2| = t+2 \\ g(t) = \sqrt{t^2 + 4 - 4t} = \sqrt{(t-2)^2} = |t-2| = t-2 \end{cases}$$

$$\rightarrow f(t) + g(t) = (t+2) + (t-2) = 2t \xrightarrow[t=\sqrt{x-4}]{t=\sqrt{x-4}} f(t) + g(t) = 2\sqrt{x-4} = a + b\sqrt{x+c}$$

$$\rightarrow \begin{cases} a=0 \\ b=2 \\ c=-4 \end{cases} \rightarrow \frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2} = \frac{1}{-2} = \left(-\frac{1}{2}\right) \quad \text{پ}$$

$$f(x) = \frac{x+2}{x^2-4x+4} = \frac{x+2}{(x-2)(x-2)} \rightarrow \text{نقاط } x=2 \text{ و } x=1 \text{ را در } D_f \text{ بررسی می‌کنیم.}$$

$$\rightarrow f(2) = \frac{2}{-4} = -\frac{1}{2} \rightarrow \left(\frac{g}{f}\right)(2) = \frac{f}{f} = 1$$

$$\rightarrow f(1) = \frac{1}{-1} = -1 \rightarrow \left(\frac{g}{f}\right)(1) = \frac{1}{-1} = -1$$

$$\frac{g}{f} = \left\{ (2, 1), (1, -1) \right\} \quad \text{پ}$$

$$\text{الف) } f(g(x)) = \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, -1\right), \left(2, 2\right), \left(-\frac{1}{2}, 6\right) \right\}$$

$$\text{ب) } f(x^2) = \left\{ (1, 2), (-1, 2), (\sqrt{3}, -1), (-\sqrt{3}, -1), (2, 2), (-2, 2) \right\}$$

$$\text{ج) } 2g(x) + 1 = \left\{ (-2, 1), (1, 3), (3, 9), (-4, 1) \right\}$$

$$\text{د) } \frac{f}{g} = \left\{ (1, -4), (3, -1) \right\} \quad \text{پ}$$

$$h = 2f + g \rightarrow D_f \cap D_g = \{2, 3, -2\}$$

$$\begin{aligned} \rightarrow 2f(2) + g(2) = h(2) = 2 + 2 = 4 \\ \rightarrow 2f(3) + g(3) = h(3) = 2 + 3 = 5 \\ \rightarrow 2f(-2) + g(-2) = h(-2) = 0 + 6 = 6 \end{aligned} \rightarrow h = \left\{ (2, 4), (3, 5), (-2, 6) \right\} \quad \text{پ}$$

$$f(x) = \frac{\sqrt{3x-2}}{x-2} \rightarrow \begin{cases} \text{① } 3x-2 \geq 0 \rightarrow x \geq \frac{2}{3} \\ \text{② } x-2 \neq 0 \rightarrow x \neq 2 \\ \text{③ } x \neq 2 \end{cases} \rightarrow \frac{2d}{1+d-0} \rightarrow x \leq 1 \quad \text{پ}$$

$$g(x) + 1 \neq 0 \rightarrow [x] + [x] + 1 \neq 0 \rightarrow \begin{cases} x \in \mathbb{Z} \xrightarrow{[x]+[x]+1} 0+1 \neq 0 \checkmark \\ x \notin \mathbb{Z} \xrightarrow{[x]+[x]+1} -1+1 = 0 \times \end{cases} \rightarrow x \in \mathbb{Z} \quad \text{پ}$$

$$\rightarrow D = \text{①} \cap \text{②} \rightarrow D = \left\{ 0, 1, 3 \right\}$$

$$f(x) = \sqrt{ax - x^2} - F \rightarrow ax - x^2 - F \geq 0 \rightarrow x^2 - ax + F \leq 0$$

$(x-1)(2x-1) \leq 0$
 $\rightarrow \frac{1}{2} \leq x \leq 1$

$$D_f \cap D_g = \{1, 2, F\}$$

$$\left. \begin{aligned} \rightarrow x=1 &\rightarrow f(1)=0 \rightarrow h(1)=h(1)=\frac{0}{1}=0 \\ \rightarrow x=2 &\rightarrow f(2)=\sqrt{F} \rightarrow h(2)=\frac{\sqrt{F}}{2} \\ \rightarrow x=F &\rightarrow f(F)=0 \rightarrow h(F)=\frac{0}{F}=0 \end{aligned} \right\} \rightarrow h(x) = \left\{ (1,0), \left(2, \frac{\sqrt{F}}{2}\right), (F,0) \right\}$$

$$\text{a) } y = x^3 - 3x^2 + 3x \rightarrow y = (x-1)^3 + 1 \rightarrow (x-1)^3 = y-1 \rightarrow x-1 = \sqrt[3]{y-1} \rightarrow x = \sqrt[3]{y-1} + 1$$

$$\rightarrow R_f = \mathbb{R}$$

$$\text{b) } y = \frac{1}{x^2 - 2x} \rightarrow yx^2 - 2yx = 1 \rightarrow yx^2 - 2yx - 1 = 0 \rightarrow \Delta = 4y^2 - 4y \geq 0$$

$$\rightarrow y^2 - y \geq 0 \rightarrow y(y-1) \geq 0 \rightarrow \frac{y-1}{y} \geq 0 \rightarrow R_f = (-\infty, -1] \cup [0, +\infty)$$

$$\text{a) } y = x^2 - 4x + 6 \rightarrow x_{\min} = \frac{b}{2a} = \frac{4}{2} = 2 \rightarrow y_{\min} = 2^2 - 4 \cdot 2 + 6 = -2 \rightarrow R_f = [-2, +\infty)$$

$$\text{b) } y = -2x^2 + 9x + 3 \rightarrow x_{\max} = \frac{b}{2a} = \frac{-9}{-4} = \frac{9}{4} \rightarrow y_{\max} = -2 \left(\frac{9}{4}\right)^2 + 9 \left(\frac{9}{4}\right) + 3 = 12 \rightarrow R_f = (-\infty, 12]$$

$$\text{c) } y = \sqrt{x^2 - 4x - 3} \rightarrow x_{\min} = \frac{b}{2a} = \frac{4}{2} = 2 \rightarrow y_{\min} = \sqrt{2^2 - 4 \cdot 2 - 3} = -1 \rightarrow R_f = \sqrt{[-1, +\infty)} = [0, +\infty)$$

$$\text{d) } y = \sqrt{4x - x^2} \rightarrow x_{\max} = \frac{b}{2a} = \frac{-4}{-2} = 2 \rightarrow y_{\max} = \sqrt{4 \cdot 2 - 2^2} = 2 \rightarrow R_f = \sqrt{[-5, 9]} = [0, 3]$$

$$\text{a) } y = x^2 - 2x^2 + 2x + 1 \rightarrow R_f = \mathbb{R}$$

$$\text{b) } y = x^2 - 2x^2 + 2x + 1 \rightarrow R_f = \mathbb{R}$$

$$\text{c) } y = \sqrt{x^2 - 4x + 1} \rightarrow R_f = \sqrt{\mathbb{R}} \rightarrow R_f = [0, +\infty)$$

$$\text{d) } y = (x^2 - 2x^2 + 2x + 1)^2 \rightarrow R_f = \mathbb{R}^+ \rightarrow R_f = [0, +\infty)$$

$$\text{a) } y = \frac{2x+1}{x-1} \rightarrow R_f = \mathbb{R} - \{1\} \rightarrow R_f = \mathbb{R} - \{1\}$$

$$\text{b) } y = \frac{2x+1}{2x+1} \rightarrow R_f = \mathbb{R} - \{1\}$$

$$\rightarrow R_f = \mathbb{R} - \{1\}$$