

$$\begin{aligned} \left. \begin{aligned} t &= \sqrt{x-4} \\ x &= t^2+4 \end{aligned} \right\} \rightarrow \begin{cases} f(t) = \sqrt{t^2+4+t} = \sqrt{(t+1)^2} = |t+1| = t+1 \\ g(t) = \sqrt{t^2+4-t} = \sqrt{(t-1)^2} = |t-1| = t-1 \end{cases} \\ \rightarrow f(t) + g(t) = (t+1) + (t-1) = 2t \xrightarrow[t=\sqrt{x-4}]{t=\sqrt{x-4}} f(t) + g(t) = 2\sqrt{x-4} = a+b\sqrt{x+c} \\ \rightarrow \begin{cases} a=0 \\ b=2 \\ c=-4 \end{cases} \rightarrow \frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2} = \frac{-1}{2} = \left(-\frac{1}{2}\right) \end{aligned}$$

$$\begin{aligned} f(x) = \frac{x+1}{x^2-4x+4} = \frac{x+1}{(x-2)(x-1)} \rightarrow \text{نقاط } x=3 \text{ و } x=1 \text{ را در } D_f \text{ قرار می‌دهیم} \\ \rightarrow f(0) = \frac{1}{-4} = -\frac{1}{4} \rightarrow \left(\frac{g}{f}\right)(0) = \frac{f}{g} = \frac{1}{-1} = -1 \\ \rightarrow f(1) = \frac{2}{-3} = -\frac{2}{3} \rightarrow \left(\frac{g}{f}\right)(1) = \frac{1}{-2} = -\frac{1}{2} \\ \rightarrow \frac{g}{f} = \left\{ (0, -1), \left(1, -\frac{1}{2}\right) \right\} \end{aligned}$$

$$الف) f(g(x)) = \left\{ \left(\frac{1}{2}, 1\right), \left(\frac{3}{2}, -1\right), \left(2, 1\right), \left(-\frac{1}{2}, -1\right) \right\}$$

$$ب) f(x^2) = \left\{ (1, 1), (-1, 1), (\sqrt{3}, -1), (-\sqrt{3}, -1), (2, 1), (-2, 1) \right\}$$

$$ج) 2g(x)+1 = \left\{ (-2, 1), (1, 3), (3, 1), (-1, 3) \right\}$$

$$د) \frac{f}{g} = \left\{ (1, -4), (3, -1) \right\}$$

$$\begin{aligned} h = f+g \rightarrow D_f \cap D_g = \{2, 3, -2\} \\ \rightarrow \left. \begin{aligned} 2f(x) + g(x) &= h(x) = 2+3=5 \\ 2f(-2) + g(-2) &= h(-2) = 2+3=5 \\ 2f(1) + g(1) &= h(1) = 0+1=1 \end{aligned} \right\} \rightarrow h = \left\{ (2, 5), (3, 5), (-2, 1) \right\} \end{aligned}$$

$$f(x) = \frac{\sqrt{3x-x}}{x-2} \rightarrow \begin{cases} ① 3x-x \geq 0 \rightarrow x \geq 0 \\ ② \sqrt{3x-x} \geq 0 \rightarrow \sqrt{3x-x} \geq x \rightarrow 3x-x \leq x^2 \rightarrow x(2-x) \leq 0 \\ ③ x \neq 2 \end{cases} \rightarrow \frac{2d}{1+d-0} \rightarrow x \leq 1$$

$$\begin{aligned} g(x)+1 \neq 0 \rightarrow [x]+[x]+1 \neq 0 \rightarrow \begin{cases} x \in \mathbb{Z} \xrightarrow{[x]+[x]+1=0+1 \neq 0} \checkmark \\ x \notin \mathbb{Z} \xrightarrow{[x]+[x]+1=-1+1=0} \times \end{cases} \rightarrow x \in \mathbb{Z} \text{ (II)} \\ \rightarrow D = \text{I} \cap \text{II} \rightarrow D = \{0, 1, 3\} \end{aligned}$$

$f(x) = \sqrt{ax - x^2} - x \rightarrow ax - x^2 \geq 0 \rightarrow x^2 - ax \leq 0$ $D_f \cap D_g = \{0, 1, 4\}$ $\rightarrow x=1 \rightarrow f(1)=0 \rightarrow h(1)=h(1)=\frac{0}{1}=0$ $\rightarrow x=2 \rightarrow f(2)=\sqrt{4} \rightarrow h(2)=\frac{\sqrt{4}}{2}$ $\rightarrow x=4 \rightarrow f(4)=0 \rightarrow h(4)=\frac{0}{4}=0$	$(x-1)(2x-4) \leq 0$ $\rightarrow \frac{x-1}{2x-4} \leq 0$ $\rightarrow \frac{x-1}{x-2} \leq 0$ $\rightarrow 1 \leq x \leq 2$ $(\frac{\sqrt{4}}{2})$ $h(x) = \{(1,0), (2, \frac{\sqrt{4}}{2}), (4,0)\}$	6
$\omega) y = x^3 - 3x^2 + 3x \rightarrow y = (x-1)^3 + 1 \rightarrow (x-1)^3 = y-1 \rightarrow x-1 = \sqrt[3]{y-1} \rightarrow x = \sqrt[3]{y-1} + 1$ $\rightarrow R_f = \mathbb{R}$	$x = \sqrt[3]{y-1} + 1$	
$\psi) y = \frac{1}{x^2 - 2x} \rightarrow yx^2 - 2yx = 1 \rightarrow yx^2 - 2yx - 1 = 0 \rightarrow \Delta = b^2 - 4ac \geq 0$ $\rightarrow 4y^2 + 4y \geq 0 \rightarrow 4y(y+1) \geq 0 \rightarrow \frac{y+1}{y} \geq 0$ $\rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$	$\rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$	7
$\omega) y = x^2 - 4x + 6 \rightarrow x_{min} = \frac{b}{2a} = \frac{4}{2} = 2 \rightarrow y_{min} = 2^2 - 4 \cdot 2 + 6 = -2 \rightarrow R_f = [-2, +\infty) = R_f$		
$\psi) y = -x^2 + 9x + 3 \rightarrow x_{max} = \frac{b}{2a} = \frac{-9}{-2} = \frac{9}{2} \rightarrow y_{max} = -(\frac{9}{2})^2 + 9(\frac{9}{2}) + 3 = 12 \rightarrow R_f = (-\infty, 12]$		
$\phi) y = \sqrt{x^2 - 4x - 3} \rightarrow x_{min} = \frac{b}{2a} = \frac{4}{2} = 2 \rightarrow y_{min} = \sqrt{2^2 - 4 \cdot 2 - 3} = -1 \rightarrow R_f = \sqrt{[-1, +\infty)} = [0, +\infty)$ $\rightarrow R_f = [0, +\infty)$		8
$\psi) y = \sqrt{4x - x^2} \rightarrow x_{max} = \frac{b}{2a} = \frac{-4}{-2} = 2 \rightarrow y_{max} = \sqrt{4 \cdot 2 - 2^2} = 2 \rightarrow R_f = \sqrt{(-\infty, 4]} = [0, 2]$ $\rightarrow R_f = [0, 2]$		
$\omega) y = x^2 - 2x^2 + 2x + 1 \rightarrow R_f = \mathbb{R}$		
$\psi) y = x^2 - 4x^2 + 4x + 2 \rightarrow R_f = \mathbb{R} = \mathbb{R}$		
$\phi) y = \sqrt{x^2 - 4x + 4} \rightarrow R_f = \sqrt{\mathbb{R}} \rightarrow R_f = [0, +\infty)$		9
$\psi) y = (x^2 - 4x + 4)^2 \rightarrow R_f = \mathbb{R}^+ \rightarrow R_f = [0, +\infty)$ $\rightarrow R_f = (\mathbb{R}^+)^2 = \mathbb{R}^+$		
$\omega) y = \frac{3x+1}{x-2} \rightarrow R_f = \mathbb{R} - \{2\} \rightarrow R_f = \mathbb{R} - \{2\}$ $\rightarrow R_f = \mathbb{R} - \{2\}$		
$\psi) y = \frac{3x+1}{2x+1} \rightarrow R_f = \mathbb{R} - \{2\}$ $\rightarrow R_f = \mathbb{R} - \{2\}$		10