

۲. اثبات عدلی درستی

$$f(x) = \sqrt{x+1} \sqrt{x-1}$$

$$g(x) = \sqrt{x-1} \sqrt{x+1}$$

$$f(x) \cdot g(x) = \sqrt{x+1} \sqrt{x-1} + \sqrt{x-1} \sqrt{x+1}$$

$$(f(x) + g(x))^2 = x+1 \sqrt{x-1} + x-1 \sqrt{x+1} + 2 \sqrt{x^2-1} \sqrt{x-1} = 2x+2\sqrt{x-1} - 1 = 2(x-1) \sqrt{x-1} \xrightarrow{\sqrt{\quad}} f(x)g(x) = \sqrt{x(x-1)} = \sqrt{x^2-x}$$

$$\frac{a+b}{c} = \frac{0+2}{-1} = -\frac{1}{1}$$

$$\boxed{a=0} \quad \boxed{b=2} \quad \boxed{c=-1}$$

$$a+b \sqrt{m+c}$$

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$$f(x) = \frac{x+2}{(x-1)(x-2)} \rightarrow D_f = \mathbb{R} - \{1, 2\}$$

$$g(x) = \{(0, 4), (1, 0), (2, 1), (3, 5)\} \rightarrow D_g = \{0, 1, 2, 3\}$$

$$\frac{g}{f} = \{(0, 4), (2, 4)\}$$

$$\begin{aligned} x &\rightarrow 0 & y &\rightarrow 1 \\ x &\rightarrow \frac{0+2}{-1-2} = \frac{2}{-3} & y &\rightarrow \frac{4}{-1-2} = -\frac{4}{3} \end{aligned}$$

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$$f(x) = \left\{ \left(\frac{1}{2}, 2 \right), \left(\frac{3}{2}, -1 \right), \left(2, 1 \right), \left(\frac{5}{2}, 4 \right) \right\}$$

$$f(x) = \left\{ (\pm 1, 2), \left(\pm \sqrt{3}, -1 \right), (\pm 2, 2), \left(\pm \sqrt{5}, 4 \right) \right\} \rightarrow \left\{ (-1, 2), (1, 2), (\sqrt{3}, -1), (-\sqrt{3}, -1), (-2, 2), (2, 2) \right\}$$

$$y \cdot g(x) + 1 = \left\{ (-2, 1), (1, 2), (3, 4), (-1, 1) \right\}$$

$$\frac{y \cdot f}{g} = \left\{ (1, -5), (3, -1), \left(\frac{5}{2}, 4 \right) \right\}$$

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$$h = 2f + g = \left\{ (2, 6), (3, 6), (-2, 6) \right\}$$

$$x \in \mathbb{R}$$

$$x(1) = 3$$

$$x(0) = 4$$

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$$D_f \Rightarrow \begin{cases} \sqrt{x} > 0 \Rightarrow x > 0 \\ \sqrt{x} - x > 0 \Rightarrow \sqrt{x} > x \xrightarrow{\sqrt{\quad}} \sqrt{x} > x^2 \Rightarrow x - x^2 < 0 \Rightarrow x(x-1) < 0 \\ x - 2 \neq 0 \Rightarrow x \neq 2 \end{cases} \Rightarrow D_f = [0, 1] - \{2\}$$

$$D\left(\frac{f}{g}\right)(x) = \{0, 1, 2\}$$

$$y \cdot g(x) + 1 \rightarrow \begin{cases} 1 & n \in \mathbb{Z} \\ 0 & n \notin \mathbb{Z} \end{cases}$$

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$$h(x) = \left[(1,0), (x, \frac{x}{x+1}), (x,0) \right]$$

$$A(x) = \sqrt{(x-1)(x+1)}$$

$$g(x) = \left[(1,1), (x,1), (x,0) \right]$$

$$0 \rightarrow \sqrt{x-1} = \sqrt{x}$$

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$$a) y = x^2 - 2x + 1 + 1 \Rightarrow y = (x-1)^2 + 1 \Rightarrow (x-1)^2 = y-1 \Rightarrow x-1 = \sqrt{y-1} \Rightarrow x = 1 + \sqrt{y-1} \quad R_y = \mathbb{R}$$

$$b) yx^2 - yx - 1 = 0 \xrightarrow{\Delta \geq 0} f_y^2 + f_y \geq 0 \Rightarrow f_y(y+1) \geq 0 \Rightarrow \frac{1}{0} \frac{0}{0} \quad D_y = \mathbb{R} \setminus (-1,0) \quad \ominus$$

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$$a) x_s = \frac{y}{y} = 1 \Rightarrow y_s = y^2 - 4x + 1 = 4 \quad a > 0 \Rightarrow R_y = [4, +\infty)$$

$$b) x_s = \frac{y}{-y} = -1 \Rightarrow y_s = -(\frac{y^2}{-1}) + 4(-1) + 1 = 12 \quad a < 0 \Rightarrow R_y = (-\infty, 12]$$

$$c) x_s = \frac{y}{y} = 1 \Rightarrow y_s = 1 - 1 - 5 = -5 \quad a > 0 \Rightarrow R_y = [-5, +\infty)$$

$$d) x_s = \frac{-y}{-y} = 1 \Rightarrow y_s = 1 - 1 - 9 = -9 \quad a < 0 \Rightarrow R_y = (-\infty, -9]$$

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$$a) R_y = \mathbb{R}$$

$$b) R_y = \mathbb{R}$$

$$c) R_y = \sqrt{12} \in [0, +\infty)$$

$$d) R_y = \mathbb{R}$$

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$$a) y = \frac{x+1}{x-1} \quad R_y = \mathbb{R} \setminus \{1\}$$

$$b) y = \frac{x+1}{x+1} \quad R_y = \mathbb{R} \setminus \{1\}$$

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