

فرهام علی دوست ۲. آزمون عالی نوشتی ۱

$f(n) = \sqrt{n+4} \sqrt{n-4}$ $n \geq 8$ $f(n) + g(n) = a + b\sqrt{n+c}$
 $g(n) = \sqrt{n-4} \sqrt{n-4}$
 $t = \sqrt{n-4} \Rightarrow t^2 = n-4 \Rightarrow n = t^2 + 4 \Rightarrow \begin{cases} f(n) = \sqrt{t^2+4+4t} = \sqrt{(t+2)^2} = |t+2| \\ g(n) = \sqrt{t^2+4-4t} = \sqrt{(t-2)^2} = |t-2| \end{cases}$
 $n \geq 8 \Rightarrow n-4 \geq 4 \Rightarrow t = \sqrt{n-4} \geq \sqrt{4} = 2 \Rightarrow t \geq 2 \Rightarrow \begin{cases} t+2 \geq 0 \Rightarrow |t+2| = t+2 \\ t-2 \geq 0 \Rightarrow |t-2| = t-2 \end{cases}$
 $f(n) + g(n) = (t+2) + (t-2) = 2t \xrightarrow{t=\sqrt{n-4}} f(n) + g(n) = 2\sqrt{n-4}$
 $\Rightarrow 2\sqrt{n-4} = a + b\sqrt{n+c} \Rightarrow 0 + 2\sqrt{n-4} \Rightarrow a=0, b=2, c=-4$
 $\frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2}$

$f(n) = \frac{n+2}{n^2+4n+3}$ $g(n) = \{(2,1) (1,0) (3,5) (0,4)\}$ $\frac{g}{f} = ?$
 $D_{\frac{g}{f}} = D_g \cap D_f - \{n \mid f(n) = 0\}$ $D_{g(n)} = \{2, 1, 3, 0\}$ $D_f \Rightarrow n^2+4n+3 \neq 0 \Rightarrow (n+1)(n+3) \neq 0 \Rightarrow n \neq -1, -3$
 $\Rightarrow D_f = \mathbb{R} - \{-1, -3\}$ $f(n) = 0 \Rightarrow n+2=0 \Rightarrow n=-2 \Rightarrow D_g \cap D_f = \{0, 2\}$
 $n=0 \Rightarrow g(0)=4$ $f(0) = \frac{0+2}{0+0+3} = \frac{2}{3} \Rightarrow \left(\frac{g}{f}\right)(0) = \frac{g(0)}{f(0)} = \frac{4}{\frac{2}{3}} = 6 = g(0) \cdot f(0)$ $n=2 \Rightarrow g(2)=1$ $f(2) = -4$
 $\left(\frac{g}{f}\right)(2) = \frac{g(2)}{f(2)} = \frac{1}{-4} = -\frac{1}{4} = g(2) \cdot f(2) \Rightarrow \frac{g}{f} = \{(0,6) (2, -\frac{1}{4})\}$

$f(n) = \{(1,2) (3,-1) (4,2) (-1,6)\}$ $g(n) = \{(1,-2,3) (1,0,-1) (3,2) (-1,0)\}$
 ۱) $f(2n) \rightarrow f(2n) = \{(1,2) (3,-1) (4,2) (-1,6)\} \rightarrow$ اینجا باید دقت کرد و باید متوجه شد که اینها را باید جدا کرد
 ۲) $f(n^2)$ $n^2=1 \Rightarrow n=1 \vee n=-1$ $n^2=9 \Rightarrow n=3 \vee n=-3$ $n^2=4 \Rightarrow n=2 \vee n=-2$ $n^2=1 \Rightarrow n=1 \vee n=-1$
 $\Rightarrow f(n^2) = \{(1,2) (3,-1) (4,2) (-1,6)\} \cup \{(1,2) (3,-1) (4,2) (-1,6)\} \cup \{(1,2) (3,-1) (4,2) (-1,6)\}$
 $\Rightarrow f(n^2) = \{(1,2) (3,-1) (4,2) (-1,6)\}$
 ۳) $2f$ $D_f \cap D_g = \{1, 3, 4, -1\} \cap \{1, 3, 2, -1\} = \{1, 3, -1\}$
 $\Rightarrow 2f = \{(1,4) (3,-2) (-1,-12)\}$

$f(n) = \{(2,1) (3,1) (4,9) (-2,1)\}$ $g = \{(5,1) (2,3) (-2,6) (1,2) (3,3)\}$ $h = 2f + g$
 $D_f = \{2, 3, 4, -2\} \cap D_g = \{5, 2, -2, 1, 3\} \Rightarrow D_h = \{2, 3, -2\}$
 $n=2 \Rightarrow h(2) = 2f(2) + g(2) = 2+3=5$ $n=3 \Rightarrow h(3) = 2f(3) + g(3) = 2+3=5$ $n=-2 \Rightarrow h(-2) = 2f(-2) + g(-2) = 2+6=8$
 $h = \{(2,5) (3,5) (-2,8)\}$

$f(n) = \frac{\sqrt{3n}-2}{n-2}$ $g(n) = [n] + [-n]$ $D = D_f \cap D_g - \{n \mid g(n) + 1 = 0\} \Rightarrow \left(\frac{f}{g+1}\right)(n)$
 $\Rightarrow 3n \geq 0 \Rightarrow n \geq 0$ $\sqrt{3n}-2 \geq 0 \Rightarrow \sqrt{3n} \geq 2 \Rightarrow 3n \geq 4 \Rightarrow n \geq \frac{4}{3}$
 $3n-2 \neq 0 \Rightarrow n \neq \frac{2}{3} \Rightarrow D_f = [\frac{4}{3}, \infty) - \{\frac{2}{3}\}$ $D_g = \mathbb{Z}$
 $\Rightarrow D = (D_f \cap D_g) \cap \mathbb{Z} = (\frac{4}{3}, \infty) \cap \mathbb{Z} - \{2\} = \{3, 4, 5, \dots\}$
 $\Rightarrow \left(\frac{f}{g+1}\right)(n) = \left\{\frac{f(n)}{g(n)+1}\right\} = \left\{\frac{f(n)}{0}\right\}$

$f(n) = \sqrt{5n-2}$, $g(n) = \{(1,3)(2,2)(4,6)(5,1)\}$ $h(n) = \frac{f(n)}{g(n)}$ -6
 $D_h = D_f \cap D_g = \{n \mid g(n) \neq 0\}$ $D_g = \{1, 2, 4, 5\}$ $5n - 2^2 - 4 \geq 0 \Rightarrow 2^2 - 5n + 4 \leq 0 \Rightarrow (n-1)(n-4) \leq 0$
 $D_h = [1, 4] \cap \{1, 2, 4, 5\} = \{1, 2, 4\} \Rightarrow n=1 \Rightarrow f(1) = \sqrt{5-2} = 1, g(1) = 3 \Rightarrow h(1) = \frac{1}{3}$
 $n=2 \Rightarrow f(2) = \sqrt{5(2)-2} = \sqrt{8} = 2\sqrt{2}, g(2) = 2 \Rightarrow h(2) = \frac{\sqrt{2}}{2}$ $(n=4 \Rightarrow f(4) = \sqrt{5(4)-2} = \sqrt{18} = 3\sqrt{2}, g(4) = 6 \Rightarrow h(4) = \frac{\sqrt{2}}{2})$
 $\Rightarrow h(n) = \left\{ \left(1, \frac{1}{3}\right) \left(2, \frac{\sqrt{2}}{2}\right) \left(4, \frac{\sqrt{2}}{2}\right) \right\}$

ا) $y = x^3 - 3x^2 + 3x - 1 \Rightarrow y - 1 = x^3 - 3x^2 + 3x - 2 \Rightarrow y - 1 = (x-1)^3$
 $\Rightarrow \sqrt[3]{y-1} = x-1 \Rightarrow x = 1 + \sqrt[3]{y-1} \Rightarrow R_f = \mathbb{R}$

$y = \frac{1}{x^2 + 2x} \Rightarrow y(x^2 + 2x) = 1 \Rightarrow yx^2 + 2xy - 1 = 0 \Rightarrow \Delta = (-2y)^2 - 4(y)(-1) \geq 0$
 $4y^2 + 4y \geq 0 \Rightarrow 4y(y+1) \geq 0$
 $\Rightarrow R_f = (-\infty, -1] \cup [0, +\infty)$

ا) $y = x^2 - 4x + 10 \rightarrow \text{ext}_{\min} \mid \frac{y}{x} = \frac{4}{2} = 2$ $4 - 8 + 10 = 6$
 $\Rightarrow R_f = [6, +\infty)$

$y = -x^2 + 6x + 3 \rightarrow \text{max} \mid \frac{y}{x} = \frac{6}{2} = 3$ $-9 + (8+3) = 12$
 $\Rightarrow R_f = (-\infty, 12]$

2) $y = \sqrt{2x^2 + 4x - 3} \rightarrow \text{min} \mid \frac{y}{x} = 2$ $4 - 8 - 3 = -7$
 $\Rightarrow R_f = \sqrt{[-7, +\infty)} = [0, +\infty) = R_f$

3) $y = \sqrt{6x - x^2} \rightarrow \text{max} \mid \frac{y}{x} = \frac{6}{2} = 3$ $6 \times 3 - 9 = 9$
 $\Rightarrow R_f = [-\infty, 9] = R_f = [0, 3]$

ا) $y = x^3 - 5x^2 + 2x + 1 \Rightarrow R_f = \mathbb{R}$

2) $y = x^7 - 6x^4 + 3x + 2 \Rightarrow R_f = \mathbb{R}$

3) $y = \sqrt{2x^7 + 6x^2 + 5x + 1} \Rightarrow R_f = \sqrt{\mathbb{R}} \Rightarrow R_f = [0, +\infty)$

4) $y = (x^5 - 4x^2 + 3x + 1)^2 \Rightarrow R_f = [0, +\infty)$

ا) $y = \frac{3x+1}{x-2} \Rightarrow R_f = \mathbb{R} - \{3\}$

2) $y = \frac{2x+1}{x+1} \Rightarrow R_f = \mathbb{R} - \{2\}$