

۲. اثبات بی ندرستی

$x + 4\sqrt{x-4} = (\sqrt{x-4} + 2)^2 \rightarrow f(x) = \sqrt{(\sqrt{x-4} + 2)^2} \rightarrow f(x) = \sqrt{x-4} + 2$ $x - 4\sqrt{x-4} = (\sqrt{x-4} - 2)^2 \rightarrow g(x) = \sqrt{x-4} - 2 \rightarrow \sqrt{x-4} \geq 2 \rightarrow g(x) = \sqrt{x-4} - 2$ $f(x) + g(x) = 2\sqrt{x-4} \rightarrow a + b\sqrt{x+c}$ $a=0 \quad b=2 \quad c=-4$ $\frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2}$	۱
$f(x) = \frac{x+2}{x^2-x+3} \rightarrow f(2) = \frac{4}{4-2+3} = \frac{4}{5} \neq -2, f(1) = \frac{3}{1-1+3} = \frac{3}{3} = 1 \neq 0, f(3) = \frac{5}{9-3+3} = \frac{5}{9} \neq 0, f(0) = \frac{2}{0-0+3} = \frac{2}{3}$ $g(x) = \{(2,1), (1,0), (3,5), (0,4)\}$ $f(x) = \{(2,-1), (0, \frac{2}{3})\}$ $g \circ f = \{(2, -\frac{1}{3}), (0, \frac{2}{3})\}$	۲
الف) $f(x) \rightarrow \{(\frac{1}{2}, 2), (\frac{3}{4}, -1), (2, 2), (-\frac{1}{2}, 9) \}$ $x^2=1 \rightarrow x=\pm 1$ $x^2=3 \rightarrow x=\pm\sqrt{3}$ $x^2=-1 \rightarrow x=\pm i$ ب) $f(x) \rightarrow \{(-1, 2), (1, 2), (2, 2), (-2, 2), (\sqrt{3}, -1), (-\sqrt{3}, -1)\}$ $x^2=1 \rightarrow x=\pm 1$ $x^2=3 \rightarrow x=\pm\sqrt{3}$ $x^2=-1 \rightarrow x=\pm i$ ج) $2g(x)+1 \rightarrow \{(-2, 19), (1, 3), (3, 9), (-1, -1)\}$ $x=-2 \rightarrow 2(2)^2+1=9, x=3 \rightarrow 2(3)^2+1=19, x=1 \rightarrow 2(1)^2+1=3, x=-1 \rightarrow 2(-1)^2+1=3$ د) $\frac{f}{g} = \{(1, 2), (3, -1), (2, 2), (-1, 9)\}$ $\frac{f}{g} = \{(1, -2), (3, -1)\}$	۳
$f(x) = \{(2,1), (3,1), (4,9), (-2,5)\}$ $g = \{(5,1), (2,3), (-2,6), (-1,2), (3,3)\}$ $2f = \{(2,2), (3,2), (4,18), (-2,10)\}$ $2f+g = \{(2,5), (-2,9), (3,5)\}$	۴
$f(x) = \frac{\sqrt{3x-x^2}}{x-2} \quad (\frac{f}{g+1})(x) \rightarrow \frac{f(x)}{g(x)+1}$ $g(x) = [x] + [-x]$ $[x] = x \quad [-x] = -x \Rightarrow \text{اگر صحیح باشد}$ $g(x) = 0 \rightarrow g(x)+1 = 1 \neq 0$ $[x] + [-x] = -1 \Rightarrow \text{اگر صحیح نباشد}$ $g(x)+1 = 0 \Rightarrow \text{مقرّر نیست}$ $\sqrt{3x-x^2} \geq 0 \Rightarrow 3x-x^2 \geq 0 \Rightarrow x(3-x) \geq 0 \Rightarrow 0 \leq x \leq 3$ $x-2 \neq 0 \Rightarrow Df = [0, 3] - \{2\}$ $\Rightarrow x \in \mathbb{Z} \mid x \in \{0, 1, 3\}$	۵

$f(x) = \sqrt{2x - x^2} - f \Rightarrow 2x - x^2 - f \geq 0 \rightarrow -x^2 + 2x - f \geq 0 \rightarrow x^2 - 2x + f \leq 0 \rightarrow (x-1)(x-f) \leq 0$ $1 \leq x \leq f \rightarrow x \in [1, f] \rightarrow f(1) = \sqrt{2-1-f} = 0 \rightarrow g(1) = 1$ $\frac{f(1)}{g(1)} = 0$, $f(2) = \sqrt{4-4-f} = \sqrt{-f}$, $g(2) = 2 \Rightarrow \frac{f(2)}{g(2)} = \frac{\sqrt{-f}}{2}$ $f(3) = \sqrt{6-9-f} = 0 \rightarrow g(3) = 3 \Rightarrow \frac{f(3)}{g(3)} = 0$ $\frac{f(x)}{g(x)} = \left\{ (1, 0), (2, \frac{\sqrt{-f}}{2}), (3, 0) \right\}$	6
الف) $y = x^3 - 3x^2 + 2x \rightarrow (x-1)^3 + 1 \rightarrow y = (x-1)^3 + 1 \rightarrow R_f = \mathbb{R}$ ب) $y = \frac{1}{x^2 - 2x} \rightarrow x^2 - 2x = (x-1)^2 - 1 \rightarrow y = \frac{1}{(x-1)^2 - 1}$ $y(x^2 - 2x) = 1 \rightarrow x^2 - 2x - \frac{1}{y} = 0 \rightarrow \Delta = (-2)^2 - 4(1)(-\frac{1}{y}) \rightarrow \Delta = 4 + \frac{4}{y} \geq 0$ $1 + \frac{1}{y} \geq 0 \rightarrow y \neq 0 \rightarrow y \in (-\infty, 0) \cup [1, +\infty) \rightarrow R_f = (-\infty, 0) \cup [1, +\infty)$	7
الف) $y = x^2 - 4x + 10 \rightarrow \min \quad -\frac{b}{2a} = \frac{4}{2} = 2 \rightarrow (2)^2 - 4(2) + 10 = 2 \rightarrow R_f = [2, +\infty)$ ب) $y = -x^2 + 4x + 3 \rightarrow \max \quad -\frac{b}{2a} = \frac{-4}{-2} = 2 \rightarrow -(2)^2 + 4(2) + 3 = 7 \rightarrow R_f = (-\infty, 7]$ ج) $y = \sqrt{x^2 - 4x - 3} \rightarrow \min \quad -\frac{b}{2a} = \frac{4}{2} = 2 \rightarrow (2)^2 - 4(2) - 3 = -7 \rightarrow R_f = [0, +\infty)$ د) $y = \sqrt{9x - x^2} \rightarrow \max \quad -\frac{b}{2a} = \frac{-9}{-2} = 4.5 \rightarrow 18 - 9 = 9 \rightarrow R_f = [0, 3]$	8
الف) $y = x^3 - 2x^2 + 2x + 1 \rightarrow R_f = \mathbb{R}$ ب) $y = x^3 - 9x^2 + 3x + 2 \rightarrow R_f = \mathbb{R}$ ج) $y = \sqrt{x^3 - 4x^2 + 2x + 1} \rightarrow R_f = [0, +\infty)$ د) $y = (x^3 - 4x^2 + 3x + 1)^3 \rightarrow R_f = [0, +\infty)$	9
الف) $y = \frac{x+1}{x-1} \rightarrow R_f = \mathbb{R} - \{1\}$ ب) $y = \frac{x+1}{x+1} \rightarrow R_f = \mathbb{R} - \{-1\}$	10