

$x + 4\sqrt{x-4} = (\sqrt{x-4} + 2)^2 \rightarrow f(x) = \sqrt{(\sqrt{x-4} + 2)^2} \rightarrow f(x) = \sqrt{x-4} + 2$ $x - 4\sqrt{x-4} = (\sqrt{x-4} - 2)^2 \rightarrow g(x) = \sqrt{x-4} - 2 \rightarrow \sqrt{x-4} \geq 2 \rightarrow g(x) = \sqrt{x-4} - 2$ $f(x) + g(x) = 2\sqrt{x-4} \rightarrow a + b\sqrt{x+c}$ $a=0, b=2, c=-4$ $\frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2}$	1
$f(x) = \frac{x+2}{x^2-x+3} \rightarrow f(2) = \frac{4}{4-2+3} = \frac{4}{5}, f(1) = \frac{3}{1-1+3} = \frac{3}{3} = 1, f(3) = \frac{5}{9-3+3} = \frac{5}{9}, f(0) = \frac{2}{3}$ $g(x) = \{(2,1), (1,0), (3,5), (0,2)\}$ $f(x) = \{(2,-1), (0, \frac{2}{3})\}$ $g \circ f = \{(2, -\frac{1}{3}), (0, \frac{2}{9})\}$	2
الف) $f(x) \rightarrow \{(\frac{1}{2}, 2), (\frac{2}{3}, -1), (2, 2), (-\frac{1}{2}, 9) \}$ $2x=1 \rightarrow x=\frac{1}{2}, 2x=3 \rightarrow x=\frac{3}{2}$ $2x=4 \rightarrow x=2, 2x=-1 \rightarrow x=-\frac{1}{2}$ ب) $f(x) \rightarrow \{(-1, 2), (1, 2), (2, 2), (-2, 2), (\sqrt{3}, 0), (-\sqrt{3}, -1)\}$ $x^2=1 \rightarrow x=\pm 1, x^2=-1 \rightarrow x=\pm i$ $x^2=3 \rightarrow x=\pm\sqrt{3}, x^2=4 \rightarrow x=\pm 2$ ج) $2g(x)+1 \rightarrow \{(-2, 19), (1, 3), (3, 9), (-1, -1)\}$ $x=-2 \rightarrow 2(x)^2+1=19, x=1 \rightarrow 2(1)^2+1=3$ $x=3 \rightarrow 2(3)^2+1=19, x=-1 \rightarrow 2(-1)^2+1=3$ د) $2f = \{(1, 4), (3, -2), (4, 4), (-1, 2)\}$ $2g = \{(-1, -\frac{1}{2}), (1, -\frac{1}{2}), (3, -1)\} \rightarrow 2f \circ 2g = \{(1, -\frac{1}{2}), (3, -1)\}$	3
$f(x) = \{(2,1), (3,1), (4,9), (-2,0)\}$ $g = \{(2,1), (2,3), (-2,6), (-1,2), (3,3)\}$ $2f = \{(2,2), (2,2), (4,18), (-2,0)\}$ $2f + g = \{(2,5), (-2,6), (2,5)\}$	4
$f(x) = \frac{\sqrt{3x-x^2}}{x-2} \quad (\frac{f}{g+1})(x) \rightarrow \frac{f(x)}{g(x)+1}$ $g(x) = [x] + [-x]$ $[x] = x, [-x] = -x \Rightarrow \text{اگر صحیح باشد}$ $g(x) = 0 \rightarrow g(x)+1 = 1 \neq 0$ $[x] + [-x] = -1 \Rightarrow \text{اگر صحیح نباشد}$ $g(x)+1 = 0 \Rightarrow \text{مقرّر نیست}$ $\sqrt{3x-x^2} \geq 0 \Rightarrow 3x-x^2 \geq 0 \Rightarrow x(3-x) \geq 0 \Rightarrow 0 \leq x \leq 3$ $x-2 \neq 0 \Rightarrow D_f = [0, 3] - \{2\}$ $\Rightarrow x \in \mathbb{Z} \Rightarrow D = \{0, 1, 3\}$	5

$f(x) = \sqrt{2x - x^2} - f \Rightarrow 2x - x^2 - f > 0 \rightarrow -x^2 + 2x - f > 0 \rightarrow x^2 - 2x + f < 0 \rightarrow (x-1)(x-f) < 0$ $1 < x < f \rightarrow x \in (1, f) \rightarrow f(1) = \sqrt{2-1-f} = 0 \rightarrow g(1) = 3$ $\frac{f(1)}{g(1)} = 0, f(2) = \sqrt{4-4-f} = \sqrt{-f}, g(2) = 2 \Rightarrow \frac{f(2)}{g(2)} = \frac{\sqrt{-f}}{2}$ $f(3) = \sqrt{6-9-f} = 0 \rightarrow g(3) = 4 \Rightarrow \frac{f(3)}{g(3)} = 0$ $\frac{f(x)}{g(x)} = \left\{ (1, 0), (2, \frac{\sqrt{-f}}{2}), (3, 0) \right\}$	6
الف) $y = x^3 - 3x^2 + 2x \rightarrow (x-1)^3 + 1 \rightarrow y = (x-1)^3 + 1 \rightarrow R_f = \mathbb{R}$ ب) $y = \frac{1}{x^3 - 2x} \rightarrow x^3 - 2x = (x-1)^3 - 1 \rightarrow y = \frac{1}{(x-1)^3 - 1}$ $y(x^3 - 2x) = 1 \rightarrow x^3 - 2x - \frac{1}{y} = 0 \rightarrow \Delta = (-2)^3 - 4(1)(-\frac{1}{y}) \rightarrow \Delta = 4 + \frac{4}{y} > 0$ $1 + \frac{1}{y} > 0 \rightarrow y \neq 0 \rightarrow y \in (-\infty, 0) \cup [1, +\infty) \rightarrow R_f = (-\infty, 0) \cup [1, +\infty)$	7
الف) $y = x^3 - 4x + 10 \rightarrow \min \quad -\frac{b}{2a} = \frac{4}{3} = 1.33 \rightarrow (1.33)^3 - 4(1.33) + 10 = 9 \rightarrow R_f = [9, +\infty)$ ب) $y = -x^3 + 4x^2 + 3 \rightarrow \max \quad -\frac{b}{2a} = \frac{-4}{-3} = 1.33 \rightarrow -(1.33)^3 + 4(1.33)^2 + 3 = 12 \rightarrow R_f = (-\infty, 12]$ ج) $y = \sqrt{x^3 - 4x - 3} \rightarrow \min \quad -\frac{b}{2a} = \frac{4}{3} = 1.33 \rightarrow (1.33)^3 - 4(1.33) - 3 = -1 \rightarrow R_f = [0, +\infty)$ د) $y = \sqrt{9x - x^3} \rightarrow \max \quad -\frac{b}{2a} = \frac{-9}{-3} = 3 \rightarrow 18 - 9 = 9 \rightarrow R_f = [0, 3]$	8
الف) $y = x^3 - 2x^2 + 2x + 1 \rightarrow R_f = \mathbb{R}$ ب) $y = x^3 - 4x^2 + 3x + 2 \rightarrow R_f = \mathbb{R}$ ج) $y = \sqrt{x^3 - 4x^2 + 2x + 1} \rightarrow R_f = [0, +\infty)$ د) $y = (x^3 - 4x^2 + 3x + 1)^3 \rightarrow R_f = [0, +\infty)$	9
الف) $y = \frac{x+1}{x-1} \rightarrow R_f = \mathbb{R} - \{1\}$ ب) $y = \frac{x+1}{x+1} \rightarrow R_f = \mathbb{R} - \{-1\}$	10