

الف) $\begin{cases} 2y_1^2 + x_1^2 + 1 = 0 \rightarrow 2y_1^2 = -x_1^2 - 1 \\ 2y_2^2 + x_2^2 + 1 = 0 \rightarrow 2y_2^2 = -x_2^2 - 1 \end{cases} \xrightarrow{x_1 = x_2} 2y_1^2 = -x_1^2 - 1 \rightarrow y_1^2 = -\frac{x_1^2}{2} - \frac{1}{2} \rightarrow y_1 = y_2$ ✓
تابع است.

ب) $x^2 + 4y^2 - 4x - 4y + 1 = 0 \xrightarrow{x=1} 1 + 4y^2 - 4 - 4y + 1 = 0 \rightarrow 4y^2 - 4y - 2 = 0$
 $\rightarrow (2y-2)^2 = 0 \rightarrow 2y-2=0 \rightarrow y=1$ تابع است.

الف) $x \sin y = y \sin x \xrightarrow{\sin x=0} 0=0 \Rightarrow \begin{cases} y=0 \\ y=\pi \\ y=2\pi \\ \vdots \end{cases}$ تابع نیست

ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \xrightarrow{\begin{matrix} x=1 \\ x=-1 \end{matrix}} \begin{matrix} y=1 \\ y=-1 \end{matrix}$ تابع است

الف) $\sqrt{\frac{x-1}{x-2}} > 0$ $\begin{matrix} x & | & 1 & 2 \\ \hline + & - & + & + \end{matrix}$ $D_f: (-\infty, 1) \cup (2, +\infty)$ $\sqrt{\frac{x-1}{x-2}} > 0 \rightarrow \frac{x-1}{x-2} > 0 \rightarrow \begin{matrix} x & | & 1 & 2 \\ \hline + & - & + & + \end{matrix}$ $D_f: (1, 2)$
 ب) $\sqrt{x} + \sqrt{y-1} = \sqrt{3} \rightarrow D_f: x \geq 0$

الف) $x^2 - \sqrt{x+4} = (x-1)(x-1) \xrightarrow{x=1} x=1$ $\begin{matrix} x & | & 1 & 4 \\ \hline + & - & + & + \end{matrix}$ $D_f: (-\infty, 1] \cup [4, +\infty)$
 $x^2 - 10x + 16 = (x-4)(x-1) \xrightarrow{x=4} x=4$ $\begin{matrix} x & | & 4 & 1 \\ \hline + & - & + & + \end{matrix}$ $D_f: (-\infty, 4) \cup (1, +\infty)$
 ب) $x^2 - \sqrt{x+4} = (x-1)(x-1) \xrightarrow{x=1} x=1$ $\begin{matrix} x & | & 1 & 4 \\ \hline + & - & + & + \end{matrix}$ $D_f: (-\infty, 1] \cup [4, +\infty)$
 $x^2 - 10x + 16 = (x-4)(x-1) \xrightarrow{x=4} x=4$ $\begin{matrix} x & | & 4 & 1 \\ \hline + & - & + & + \end{matrix}$ $D_f: (-\infty, 4) \cup (1, +\infty)$

الف) $x^2 - 5x + 4 \rightarrow 0$ و $\frac{1}{x} \rightarrow \begin{matrix} x & | & 1 & 4 \\ \hline + & - & + & + \end{matrix} \xrightarrow{y>0} (-\infty, 1] \cup [4, +\infty)$
 $x^2 - \sqrt{x+4} \rightarrow 0$ و $\frac{4}{x} \rightarrow \begin{matrix} x & | & 4 & 1 \\ \hline + & - & + & + \end{matrix} \xrightarrow{y>0} (-\infty, 4) \cup (1, +\infty)$
 ب) $\frac{x^2 - 5x + 4}{x^2 - \sqrt{x+4}} > 0 \Rightarrow \begin{matrix} x & | & 1 & 4 & 4 & 1 \\ \hline + & - & + & - & + & + \end{matrix} \xrightarrow{y>0} (-\infty, \frac{4}{5}) \cup (\frac{4}{5}, +\infty)$
 $\rightarrow 0$ و $(\frac{4}{5})$ $R - [\frac{4}{5}, \frac{4}{5})$

$$A \times B = (1, 2) \cup (1, 3) \cup (2, 2) \cup (2, 3) \cup (3, 2) \cup (3, 3)$$

$$B \times B = (1, 2) \cup (2, 2) \cup (2, 3) \cup (3, 2) \cup (3, 3)$$

$$A \times B - B^2 = (1, 2) \cup (1, 3)$$

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$$m^2 = m+2 \Rightarrow m^2 - m - 2 = (m-2)(m+1)$$

تابع است $\Rightarrow (1, 2) \cup (2, 3)$ $\Rightarrow m=2$ ✓ $m=-1$ ✗

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$$m^2 = m+2 \Rightarrow m^2 - m - 2 = (m-2)(m+1)$$

تابع است $\Rightarrow (-1, -2) \cup (-2, -3)$ $\Rightarrow m=-1$ ✗ $m=2$ ✓

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$$f(x) = \begin{cases} x^2 & x \leq 1 \\ ax+b & 1 < x < 2 \\ x^2 & x \geq 2 \end{cases}$$

$x=1 \Rightarrow 1-1 = a+b \Rightarrow a+b=0$
 $x=2 \Rightarrow 4 = 2a+b$
 $a=1, b=-1$
 $\Rightarrow a \times b = 1 \times (-1) = -1$

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$$f(x) = \begin{cases} x^2 & x \geq k \\ x-k & x < k \end{cases}$$

$x=k \Rightarrow k-k = f(k)$
 $k-k=0$
 $k=\frac{1}{2}$

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