

لازمی و سید
تالیف ۱۱

۱۵

امیر محمد زواری

$$f(x) = \begin{cases} x^3 - 2 & ; x > a \\ 12a - x & ; x \leq a \end{cases} \quad a^3 = 8$$

بر مبنای

(۱)

$$a^3 - 2 \geq 12a - 2 \rightarrow a^3 - 12a + 2 \geq 0 \rightarrow a^3 - 12a \geq -2 \rightarrow a(a^2 - 12) \geq -2$$

$$a^3 - 2 \geq 12a - 2 \rightarrow a^3 - 12a + 2 \geq 0 \rightarrow a^3 - 12a \geq -2 \rightarrow a(a^2 - 12) \geq -2$$

$$(a-2)^2(a+4) \geq 0 \quad \frac{-2}{-1+1+} \quad \underline{a \geq -4}$$

$$f(x) = 3x + k \quad f^{-1}(x) = \frac{x-k}{3} = y \quad \frac{x-k}{3} \rightarrow \frac{x-k}{3} = y \rightarrow x-k = 3y \rightarrow x = 3y+k$$

الف) $f(1) = 3(1) + k = 11 \rightarrow k = 8$

ب) $f(f(x)) = f(3x+k) \rightarrow 3(3x+k) + k = 9x + 4k = 9x + 32$

(۲)

A | $\frac{2a}{a}$ $f(x) = \frac{ax}{x-1} \rightarrow y = \frac{ax}{x-1} \rightarrow y(x-1) = ax \rightarrow yx - y = ax \rightarrow yx - ax = y \rightarrow x(y-a) = y \rightarrow x = \frac{y}{y-a} \rightarrow y = \frac{x}{x-a}$

(۲)

$$\frac{2a}{2a-a} = a \rightarrow \frac{2a}{a} = a \rightarrow a^2 = 2a - a^2 - 2a = 0 \rightarrow a(a-2) = 0 \rightarrow a = 0 \text{ or } a = 2$$

$$f = \{(1,2), (2,1), (3,3)\} \rightarrow f^{-1} = \{(2,1), (1,2), (3,3)\}$$

$$g = \{(1,3), (2,1), (3,2)\} \rightarrow g^{-1} = \{(3,1), (1,2), (2,3)\}$$

الف) $f \circ f^{-1} \rightarrow f(f^{-1}(x)) \rightarrow \{(1,1), (2,2), (3,3)\}$

ب) $f^{-1} \circ f = \{(1,1), (2,2), (3,3)\}$

ج) $f \circ g^{-1} = \{(1,2), (2,3), (3,1)\}$

د) $g^{-1} \circ f = \{(1,3), (2,1), (3,2)\}$

(۲)

$$f = \{(1,2), (2,1), (3,3)\} \quad g = \{(1,3), (2,1), (3,2)\} \quad h = \{(1,2), (2,3), (3,1)\}$$

$$f \circ g^{-1} = \{(1,2), (2,3), (3,1)\}$$

$$g^{-1} = \{(1,3), (2,1), (3,2)\}$$

$$\frac{(1,2) (2,3) (3,1)}{(1,3) (2,1) (3,2)}$$

$$\{(1,2), (2,3), (3,1)\}$$

(۱, ۱/۵)

(1,5)

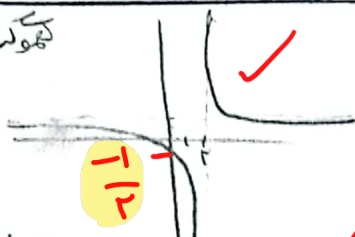
گنجه را به یک است $y = \frac{3u+1}{u-2}$

$3u - 2y = 2u + 1$

$u(y-3) = 1+2y \rightarrow u = \frac{1+2y}{y-3} \rightarrow y = \frac{2u+1}{u-3}$

سوال مفردار معلوم رو
مخواد!

محل برخورد با محورهای مختصات



$y = |u-1| - |u-3| \rightarrow [4, 6]$

$-u + 1 + u - 3 = -2$
 $u - 1 + u - 3 = 2u - 4$



$y = 2u - 4 \rightarrow 2u = y + 4 \rightarrow u = \frac{y+4}{2} \rightarrow y = \frac{u+4}{2}$

$D = [-2, 2]$

همیشه وقتی معکوس تابعی را بدست می دهی باید دامنه اش را معکوس کنی!

$f(u) = \begin{cases} u^3 + 9 & u \geq 1 \\ 2u - 1 & u \leq 0 \end{cases}$

$u^3 + 9 = u \rightarrow u = \sqrt[3]{u-9}$

$2u - 1 = y \rightarrow 2u = y + 1 \rightarrow u = \frac{y+1}{2}$

$(\frac{y+1}{2}, -1]$

$\begin{cases} y = \sqrt[3]{u-9} & u \geq 9 \\ u + \frac{1}{2} & u \leq -1 \end{cases}$

$f(u) = u^2 - \frac{(u+1)^2}{u+3}$

$f^{-1}(u) = \frac{au+b}{cu+d}$

$f^{-1}(6) = \frac{a \cdot 6 + b}{c \cdot 6 + d}$

$f^{-1}(u) = \frac{-3u-1}{u+3} \rightarrow f^{-1}(u) = \frac{-4u-2}{2u+4}$

$b = -2$

$y = \frac{-3u-1}{u+3} \rightarrow yu + 3y = -3u - 1 \rightarrow u(y+3) = -1-3y \rightarrow y = \frac{-(3u+1)}{u+3} \rightarrow a = -3, d = 3$

$6 \rightarrow \frac{-3 \cdot 6 - 1}{3 \cdot 6 + 3} = \frac{-19}{21} \rightarrow f^{-1}(-\frac{19}{21}) = 6$

$f(u) = \frac{u}{u^2+1}$

$yu^2 + y = u \rightarrow yu^2 - u + y = 0 \rightarrow \Delta = 1 - 4(y)(y) = 1 - 4y^2 \geq 0$

$1 \geq 4y^2 \rightarrow \frac{1}{2} \geq y^2 \rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$

(۱۵) سوال بررسی یک به یکی دو غیر حفاذ! خدا بقاء معلول تابع از حفاذ که به معلومش تابع نیست!

$$y = \frac{u}{u^2 + 1} \rightarrow u = \frac{y}{y^2 + 1} \rightarrow uy^2 + u = y$$

$$uy^2 - y + u = 0 \rightarrow y = \frac{1 \pm \sqrt{1 - 4u^2}}{2u} \quad |u| \leq \frac{1}{2} - \{0\}$$