مهدی کاظمی تکلیف شماره ۱۱

۱-

$f(x) = \begin{cases} x^3 - 4 \; : \; x > a \\ 12x - 20 \; ; \; x \leq a \end{cases}$

$a^3 - 4 \geq 12a - 20$

$a^3 - 12a + 16 \geq 0 \longrightarrow (a-2)^2 (a+4) \geq 0$

	-4		2	
	$-$	$+$	$+$	

الیزا مسعودی

$a \geq -4 \qquad a : [-4, +\infty)$

۲-

$f(x) = 3x + k \;,\; f^{-1}(2) = 4 \qquad 3y = 3x + k \longrightarrow y = \dfrac{x-k}{3} \longrightarrow f^{-1}(x) = \dfrac{x-k}{3}$

$f^{-1}(2) = \dfrac{2-k}{3} = 4 \longrightarrow k = -10$

الف) $f(7) = 3 \times 7 - 10 = 11$

ب) $f(f(x)) = 3(3x - 10) - 10 = 9x - 40$

۳-

$A(2a, a) \;,\; f(x) = \dfrac{ax}{x-1} \;,\; a = ?$

$y = \dfrac{ax}{x-1} \rightsquigarrow \dfrac{ay}{y-1} \longrightarrow xy - x = ay \longrightarrow ay - xy = -x \longrightarrow y(a - x) = -x$

$y = \dfrac{-x}{a-x} \longrightarrow f^{-1}(x) = \dfrac{-x}{a-x}$

$f^{-1}(2a) = \dfrac{-2a}{a - 2a} = a$

$\dfrac{2a}{a} = 2 = a$

۴-

الف) $fof^{-1} = \{(5,5)(7,7),(7,7)(6,6)\}$

ب) $f^{-1}of = \{(3,3),(4,4),(7,7),(9,9)\}$

ج) $fog^{-1} = \{(2,5),(5,6)\}$

د) $g^{-1}of = \{(2,9),(9,8)\}$

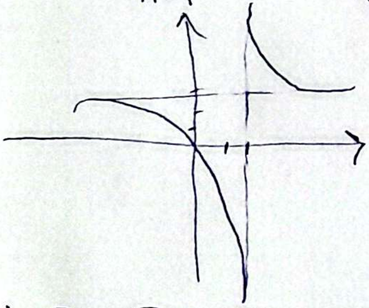
۵-

$\dfrac{h}{fog^{-1}} = ? \qquad gog^{-1} = \{(1,3),(4,8),(5,5)\}$

$\longrightarrow \{(1, \tfrac{1}{2}), (3, \tfrac{1}{2})\}$

۶-

$y = \dfrac{3x+1}{x-2} \longrightarrow yx - 2y = 3x + 1$



$3x - yx = 2y - 1$

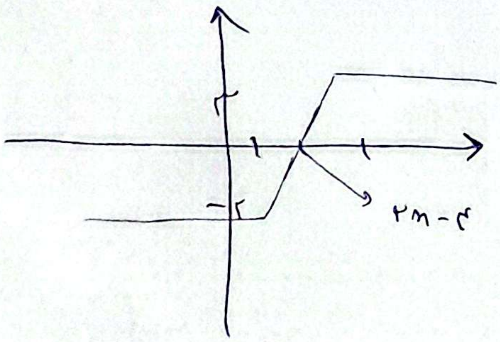
$x(3 - y) = 2y - 1$

$x = \dfrac{2y+1}{y-3}$

$y = \dfrac{3x+1}{x-3} \longrightarrow f^{-1}(x) = \dfrac{3x+1}{x-3}$

۷-

$f(x) = |x-1| - |x-3|$



$\begin{cases} 2 & x < 1 \\ 2x - 4 & 1 \leq x \leq 3 \\ -2 & x \geq 3 \end{cases}$

$f(x) = 2x - 4 \longrightarrow y = 2x - 4 \longrightarrow x = 2y - 4$

$2y = x + 4 \longrightarrow y = \dfrac{x+4}{2} \longrightarrow f^{-1}(x) = \dfrac{x+4}{2}$

$D_{f^{-1}} : [-2, 2]$

$R_{f^{-1}} : [1, 3]$

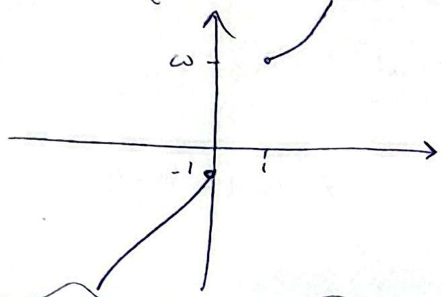
-۸

$f(x) = \begin{cases} x^3+4 \; ; & x \geq 1 \\ 4x-1 \; ; & x \leq 0 \end{cases}$

$x^3+4=y$

$y^3 = x-4$

$y = \sqrt[3]{x-4}$



$f^{-1}(x) = \begin{cases} \sqrt[3]{x-4} \; ; & \dfrac{R_{f(x)}}{x \geq 5} \\ \dfrac{x+1}{4} \; , & \dfrac{x \leq -1}{R_{f(x)}} \end{cases}$

$4x-1=y$

$4y = x+1$

$y = \frac{x+1}{4}$

$f(x) = x^2 - \frac{(x+1)^3}{x+3} \qquad f^{-1}(x) = \frac{ax+b}{2x+d} \qquad f^{-1}(b) = ? \quad b<0$

$\frac{x^2(x+3)}{x+3} - \frac{(x+1)^3}{x+3} = \frac{\cancel{x^3}+3x^2-\cancel{x^3}-1}{x+3} = \frac{3x^2-1}{x+3} = f(x)$

$y = \frac{-3x-1}{x+3} \longrightarrow f^{-1}(x) = \frac{-3x-1}{x+3} = \frac{-6x-2}{2x+6}$

$a=-6$

$b=-2$

$d=6$

$f^{-1}(-2) = \frac{10}{2} = 5$

$f(x) = \frac{x}{x^2+1}$ -۱۰

$y = \frac{x}{x^2+1} \longrightarrow x = \frac{y}{y^2+1} \rightarrow xy^2+x=y$

$xy^2+x-y=0$

$y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$