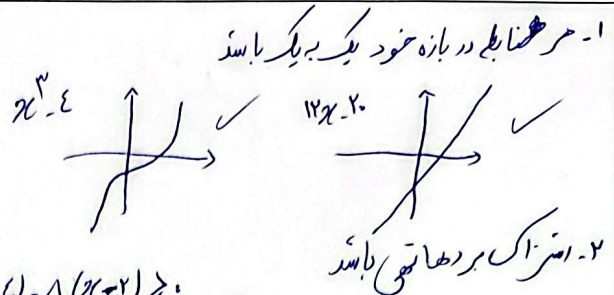


$$f(x) = \begin{cases} x^3 - 4 & ; x > a \\ 12x - 2 & ; x \leq a \end{cases}$$



$$\begin{aligned} x^3 - 4 &\geq 12x - 2 \\ \Rightarrow x^3 - 12x + 14 &\geq 0 \Rightarrow x^3 - 4x - 8x + 14 > 0 \Rightarrow x(x^2 - 4) - 8(x - 2) > 0 \\ \Rightarrow (x - 2)(x^2 - 2x - 8) &\geq 0 \Rightarrow (x - 2)^2(x + 4) \geq 0 \end{aligned}$$

$a = [-4, +\infty)$

$$\begin{aligned} f^{-1}(4) = 2 &\Rightarrow f(2) = 4 \\ f(x) = 3x + K &\Rightarrow f(2) = 12 + K \Rightarrow K = -10 \\ f(x) &= 3x - 10 \end{aligned}$$

الف) $f(7) = 3 \times 7 + (-10) = 21 - 10 = 11$

ب) $f(f(x)) = f(3x - 10) = 3 \times (3x - 10) - 10 = 9x - 30 - 10 = 9x - 40$

$$f(a) = \frac{a^2}{a-1} = 2a \Rightarrow a^2 = 2a^2 - 2a \Rightarrow a^2 - 2a = 0 \Rightarrow a(a-2) = 0$$

$$\Rightarrow \begin{cases} a=0 \rightarrow f(a)=0 \\ a=2 \rightarrow f(a)=0 \end{cases}$$

بدیهه نیست

الف) $f \circ f^{-1} = \{(5, 5), (7, 7), (2, 2), (4, 4)\}$ ب) $f^{-1} \circ f = \{(3, 3), (4, 4), (7, 7), (9, 9)\}$

$$5 \rightarrow 3 \rightarrow 5$$

$$7 \rightarrow 4 \rightarrow 7$$

$$2 \rightarrow 7 \rightarrow 2$$

$$4 \rightarrow 9 \rightarrow 4$$

$$c) f \circ g^{-1} = \{(2, 5), (5, 9)\}$$

$$2 \rightarrow 3 \rightarrow 5$$

$$5 \rightarrow 8 \rightarrow 9$$

$$5 \rightarrow 9 \rightarrow 4$$

$$3 \rightarrow 5 \rightarrow 9$$

$$4 \rightarrow 7 \rightarrow 4$$

$$7 \rightarrow 2 \rightarrow 7$$

$$9 \rightarrow 6 \rightarrow 9$$

$$d) g^{-1} \circ f = \{(3, 9), (7, 3), (9, 1)\}$$

$$3 \rightarrow 5 \rightarrow 9$$

$$7 \rightarrow 2 \rightarrow 7$$

$$9 \rightarrow 6 \rightarrow 9$$

$$f \circ g^{-1} = \{(1, 4), (3, 8), (5, 0)\}$$

$$1 \rightarrow 2 \rightarrow 4$$

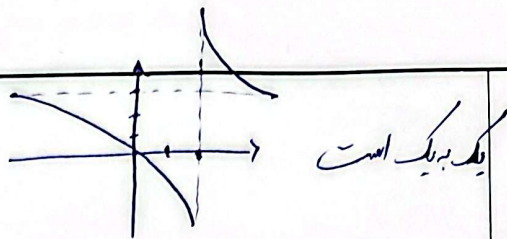
$$3 \rightarrow 4 \rightarrow 8$$

$$5 \rightarrow 6 \rightarrow 0$$

$$\frac{h}{f \circ g^{-1}} = \{(1, \frac{4}{5}), (3, \frac{8}{5}), (5, \frac{0}{5})\}$$

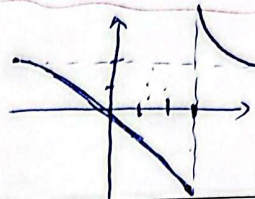
$$\frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{5}), (3, \frac{1}{5})\}$$

$$y = \frac{3x+1}{x-2} \rightarrow \begin{cases} \text{م افقی} = \frac{a}{c} = 3 \\ \text{م عمودی} = \frac{b}{c} = 2 \end{cases}$$



$$f^{-1}(x) = \frac{3x+1}{x-3} \rightarrow \begin{cases} \text{م افقی} = \frac{a}{c} = 3 \\ \text{م عمودی} = \frac{b}{c} = 2 \end{cases}$$

$x=0 \rightarrow f^{-1}(x) = -\frac{1}{3}$



$$f(x) = |x-1| - |x-3|$$

$$f^{-1}(x) = \frac{x+2}{2} = \frac{1}{2}x + 1$$

$$\begin{array}{c|c|c} x & f(x) & f^{-1}(x) \\ \hline x=1 & -2 & -1 \\ x=2 & 0 & 1 \\ x=3 & 2 & 2 \end{array}$$

$R_f = D_{f^{-1}} = [-2, 2]$

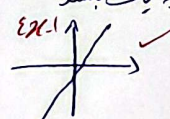
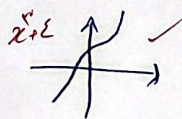
$$\begin{aligned} x=1 &\Rightarrow f(x) = -2 \\ x=2 &\Rightarrow f(x) = 0 \\ x=3 &\Rightarrow f(x) = 2 \end{aligned}$$

$a=1, b=3$

$$f(x) = \begin{cases} x^2 - 4 & ; x \geq 1 \\ 4x - 1 & ; x \leq 0 \end{cases}$$

$$\begin{aligned} R_{y_1} &= [\omega, +\infty) \\ R_{y_2} &= (-\infty, -1] \end{aligned} \Rightarrow R_{y_1} \cap R_{y_2} = \emptyset$$

$$f^{-1}(x) = \begin{cases} \sqrt{x+4} & ; x \geq \omega \\ \frac{1}{4}x + \frac{1}{4} & ; x \leq -1 \end{cases}$$



۲. اشتراک برد نمی باشد
یک بزرگ است

$$f(x) = x^2 - \frac{x^2 + 3x^2 + 3x + 1}{x+3} = \frac{x^2 + 3x^2 - x^2 - 3x^2 - 3x - 1}{x+3} \Rightarrow f(x) = \frac{-3x-1}{x+3}$$

$$\Rightarrow f^{-1}(x) = \frac{-3x-1}{x+3} = \frac{-4x-4}{2x+4} \Rightarrow \begin{cases} a = -4 \\ b = -4 \\ d = 4 \end{cases}$$

$$x = -2 \Rightarrow f^{-1}(-2) = \frac{-4(-2)-4}{2(-2)+4} = \frac{4-4}{-4+4} = \frac{0}{0} = \omega$$

$$y = \frac{x}{x^2+1} \Rightarrow yx^2 + y = x \Rightarrow yx^2 - x + y = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

$$\Rightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x} \Rightarrow f^{-1}(x) = \frac{1 \pm \sqrt{1-4x^2}}{2x} \quad f\left(\frac{1}{\omega}\right) = \frac{1}{\omega} \Rightarrow f^{-1}\left(\frac{1}{\omega}\right) = \frac{1}{\omega}$$

$$\Rightarrow f^{-1}(x) = \frac{1 + \sqrt{1-4x^2}}{2x}$$